R. H. MANN.

£4

The Association
of
Engineering and Shipbuilding Draughtsmen.

DESIGN AND CONSTRUCTION IN PRESTRESSED CONCRETE.

By J. E. GUEST, A.M.I.Struct.E., M.Soc.C.E. (France).

Published by The Association of Engineering and Shipbuilding Draughtsmen,
96 St. George's Square, London, S.W.I.

SESSION 1951-52.

Printed by Milne, Tannahill & Methven, Ltd. (T.U.), 12-14 Mill Street, Perth.

ADVICE TO INTENDING AUTHORS OF AE.S.D. PRINTED PAMPHLETS.

Pamphlets submitted to the National Technica Sub-Committee for consideration with a view to publication in this series should not exceed 9,000 to 10,000 words and about 20 illustrations making a pamphlet of about 40 to 48 pages. The aim should be the presentation of the subject clearly and concisely, avoiding digressions and redundance. Manuscripts are to be written in the third person.

Drawings for illustrations should be done either on a good plain white paper or tracing cloth, deep black Indian ink being used. For ordinary purposes they should be made about one-and-a-half times the intended finished size, and it should be arranged that wherever possible these shall not be greater than a single full page of the pamphlet, as folded pages are objectionable, although, upon occasion, unavoidable. Where drawings are made larger, involving a greater reduction, the lines should be made slightly heavier and the printing rather larger than normal, as the greater reduction tends to make the lines appear faint and the printing excessively small in the reproduction. In the case of charts or curves set out on squared paper, either all the squares should be inked in, or the chart or curve should be retraced and the requisite squares inked in. Figures should be as self-evident as possible. Data should be presented in graphical form. Extensive tabular matter, if unavoidable, should be made into appendices.

Authors of pamphlets are requested to adhere to the standard symbols of the British Standards Institution, where lists of such standard symbols have been issued, as in the case of the electrical and other industries, and also to the *British Standard Engineering Symbols and Abbreviations*, No. 560, published by the B.S.I. in 1934 at 5/-. Attention might also be given to mathematical notation, where alternative methods exist, to ensure the minimum trouble in setting up by the printer.

The value of the pamphlet will be enhanced by stating where further information on the subject can be obtained. This should be given in the form of footnotes or a bibliography, including the name and initials of the author, title, publisher, and year of publication. When periodicals are referred to, volume and page also should be given. References should be checked carefully.

Manuscripts, in the first instance, should be submitted to the Editor, *The Draughtsman*, 96 St. George's Square, London, S.W.1.

For Pamphlets, a grant of £20 will be made to the Author, but special consideration will he given in the case of much larger pamphlets which may involve more than the usual amount of preparation.

R. H. MANN.

The Association
of
Engineering and Shipbuilding Draughtsmen.

DESIGN AND CONSTRUCTION IN PRESTRESSED CONCRETE.

By J. E. GUEST, A.M.I.Struct.E., M.Soc.C.E. (France).

Published by The Association of Engineering and Shipbuilding Draughtsmen
96 St. George's Square, London, S.W.1.

SESSION 1951-52.

9/52.

PREFACE.

Information on prestressed concrete, although available in considerable quantity, is largely to be found in the journals of the engineering institutions and in technical magazines. Many of these are not easily obtainable and many are specialists' appreciations of some particular aspect of the subject.

The intention of the author in preparing this paper is to show that prestressed concrete design may be carried out by the ordinary structural engineer. Once the physical properties of the steel and concrete are defined the calculations are more simple than those for a comparable structure in reinforced concrete. The difficulties to be overcome are inherent in the complex nature of one of the component materials, concrete, and will not be eliminated by fussy computations. In our calculations we should, therefore, be guided by the words of one of the greatest engineers of our day, Robert Maillart—"simplicity is more worth striving after than the greatest precision which is naturally unobtainable."

At the same time the use of prestressed concrete involves a new approach to design in order to achieve the maximum advantages of the technique. This outlook will only come as familiarity with the methods is gained and after a careful study of the work that has already been done.

A paper of this size can only pretend to be an introduction; the examples have been confined to simple beams to avoid obscuring the basic elements of the method, but it is hoped that the reader will take advantage of the selective bibliography to further his knowledge of the subject.

The writer would like to acknowledge the helpful criticism and careful reading of the paper by Mr. S. K. Mallick, B.Sc., A.M.I.Struct.E.

J. E. GUEST.

CONTENTS.

DESIGN AND CONSTRUCTION IN PRESTRESSED CONCRETE

By J. E. Guest, A.M.I.Struct.E., M.Soc.C.E. (France)

Chapter I.

PRINCIPLES, DEVELOPMENT AND METHODS.

Why Prestress Concrete ?

The idea of applying an external force to a structural member or to a complete structure to counteract undesirable stresses set up in the member by other external forces has long been utilized by engineers; retaining walls built of separate blocks maintain their stability by the addition of a compressive force (their own weight) to a horizontal force producing tension, which the wall cannot resist. However, it is only in the last 50 years or so that the full engineering possibilities of controlling both the magnitude and direction of such forces have been realized. Prestressing, as this technique is called, may be applied to many materials by a variety of means. The principle consists in applying to a member before loading a force that causes in the member stresses of an opposing nature to those set up by the load to be supported.

Although there is the possibility of using other materials, the two that are entirely used at the moment are concrete and steel. Concrete is comparatively cheap in cost. It may easily and economically be moulded into any shape desired and it has a good compressive strength; unfortunately its strength in tension is only about 10% of its strength in compression. The realization that steel, equally strong in tension and compression, but much more costly than concrete, could be combined with the concrete so that the latter resisted the compressive forces, and the steel the tensile forces, led to the use of reinforced concrete. Here we have a composite material which utilizes as well as it may the properties of both materials to make a more economical structure than could be obtained with either material separately.

Unfortunately, reinforced concrete has a number of disadvantages, amongst them is its weight, which precludes its use on long spans except in the special form of the arch and the shell. There is also the difficulty of guarding against cracks. The reinforcement does not prevent crack formation: it reduces and distributes the cracks in such a manner that their presence is in many cases innocuous. There is, however, a paradox in the development of reinforced concrete and it is that beyond a certain

limit we cannot make further use of the advantages of the higher strength concretes and steels now being offered to us. Particularly in the case of steel, we cannot increase the stress much beyond the present limit of 27,000 p.s.i. without endangering the member, because of the widening of the cracks brought about by the increase in strain of the steel. It is in the fact that prestressed concrete enables us to use—indeed it is essential that we do use—the highest attainable stresses that makes it a more economical material compared to reinforced concrete.

Early Work on Prestressed Concrete.

Strangely enough, the idea of using pretensioned steel to bring about a state of compression in the concrete to which it was fixed, occurred to engineers at the same time, or thereabouts, as the introduction of normal reinforced concrete construction, for in the year 1888, C. F. W. Doehring took out a patent in Berlin for mortar slabs reinforced by steel wires which were prestressed.

Notwithstanding the early introduction of the idea of prestressing, progress was negligible until the work of Eugene Freyssinet was made known in the years following 1926. It is, indeed, due to Freyssinet that we owe all the original work on the principles underlying the practical application of prestressed concrete, as well as the most outstanding examples of its utilization.

The early designers had all failed because they had not realized the importance of the losses in the initial prestressing forces brought about by various relaxations occurring in the materials over a period of time. These losses are fairly constant for a given compressive stress in the concrete and do not depend to any great extent on the initial tension in the wires. Their value lies normally between 15,000 p.s.i. and 30,000 p.s.i. Thus, using as did Doehring, Mandl, Koenen, Lund, Dill and Hewett, steel of medium quality with a yield point of about 30,000 p.s.i. to 40,000 p.s.i., it was not possible to stress the wires sufficiently highly for there to be any useful force left after all the relaxations had taken place.

The investigations into the properties of concrete carried out by Freyssinet in France and by Faber and Glanville in England showed that under a compressive stress of 1500 p.s.i. a strain of about $0{\cdot}3 \times 10^{-6}$ per lb. per sq. inch might be expected, that is, there would be a strain increase of $0{\cdot}3 \times 10^{-6} \times 1500 = 450 \times 10^{-6}$ inches per inch. High tensile steel has a value for E of about 29×10^{6} p.s.i. Thus the creep in the concrete represents a loss of $450 \times 10^{-6} \times 29 \times 10^{6}$ p.s.i. $= 13{,}000$ p.s.i. in the steel. In a similar way, shrinkage of the concrete would cause a loss of one half to two-thirds of the loss due to creep. It was to be seen therefore that the use of as high an initial steel stress as possible was not only economically desirable but actually essential to the main-

tenance of that force upon which stability of the construction depended. From the time that this was appreciated by Freyssinet, prestressed concrete became a practicable form of construction.

The Effects of Prestressing.

Before going further it is advisable to ensure that we understand how the principle of prestressing is applied. For example, let us consider a simple beam freely supported. Let it be of rectangular section, $b \times h$, and suppose it to have no self weight.

Under the influence of external loading we will have for an homogeneous material a stress distribution such as that shown in Fig. 1, the extreme fibre stresses being, say $\pm$ 100 p.s.i.

Let the maximum fibre stresses for the material of which the beam is made be 100 p.s.i. in compression and zero in tension. The stress diagram shown is then impossible: under the loading the beam will break.

However, before loading the beam let us apply to its ends a force F at such a distance from the centroid of the section that we obtain a stress distribution as shown in Fig. 2.

By inspection we see that F is equal to $bhf/2$, and is applied at the lower "middle third" limit, *i.e.*, at $h/6$ from the centre. Thus by prestressing we cause a compressive stress in the bottom of the beam equal in intensity to the tensile stress brought about by the application of load. At the top fibre there is zero prestress and eventually a compressive stress under load. The superposition of these two sets of stresses results finally in there being zero stress in the bottom fibres and 100 p.s.i. compression in the top fibres, Fig. 3.

To go one step further we will assume that the beam has a dead weight, which before the beam is prestressed is carried by the formwork. As the prestressing forces are brought into play they cause the beam to "hog;" in its turn this upward deflection is resisted by the self weight of the beam, the latter causing stresses to be set up in opposition to those caused by the prestressing and brought into play as the prestressing takes place. We are, therefore, not concerned with the stresses due to prestressing by themselves but with the combination of these and the dead load stresses.

Let us assume that in our previous example the dead load stresses are $\pm$ 50 p.s.i.; then we may have the combinations of stress diagrams shown in Fig. 4.

In stage I we have on our final stress diagram the same values as we had formerly for the prestressing alone. Consequently, when under live load, the beam is stressed to no higher a value than before. It will also be seen that F has the same value—we have, in fact, carried the live load of our previous example and in addition a dead load equivalent to one half the live load (assuming both loads uniformly distributed) by a beam of the same size and with the

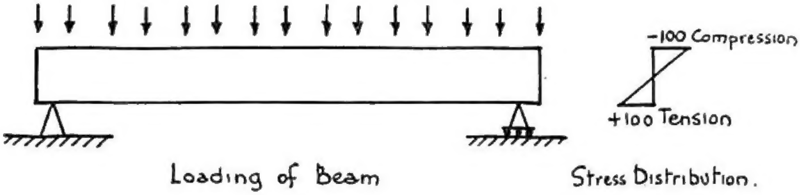


FIGURE 1.

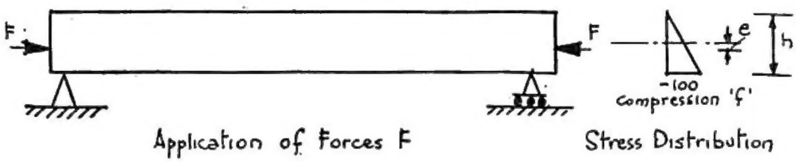


FIGURE 2.

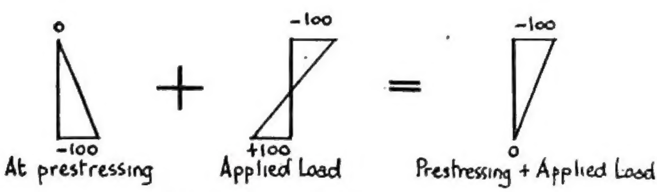


FIGURE 3.

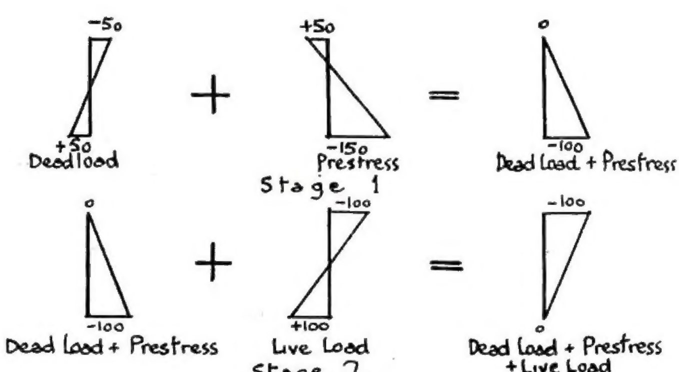


FIGURE 4.

same prestressing force. This has been done merely by displacing the point of application of the force F and it illustrates one of the most important advantages of prestressed concrete compared to normal reinforced concrete construction, which is that up to a certain limit we may carry the permanent dead load as well as the applied load on a member dimensioned entirely by the value of the applied load. It should be noted particularly that by dead load we mean the permanent load which exists when the prestress is applied. Normally that is the self weight of the member and it does not include any of the dead weight of construction to be applied after prestressing is completed.

From the equations given in Chapter III, relating to beams bending in one direction only, it is possible to determine the theoretical limit for the moment due to the permanent dead load M_{DL}, which may be carried by a section of any shape, without increase in the dimensions over those required to resist the applied load moment.

If the section has the properties given in Fig. 5 and if the maximum applied load moment that this section can resist is M_{SL}, then,

$$\frac{M_{DL}}{M_{SL}} = \frac{v_1\,(vv_1 - \rho^2)}{\rho^2\,h}$$

which for a rectangular section equals 1.

This expression assumes that the prestressing force is applied at the lower extreme fibre of the section and that there is no reduction in the force due to creep or shrinkage. In practice, therefore, the ratio will be less than indicated above and for a rectangular section will lie between 0·65 and 0·9.

It should be noted that as we move away from the point of maximum moment it is necessary to reduce the value of the stresses due to prestressing in order not to overstress the material. In most practical cases, as will be described later, the prestress is applied by means of a stretched tendon which we may bend upwards towards the supports, thus reducing the eccentricity and consequently the stresses due to the tensioning force. This has advantages in addition in that the vertical component of the tangent to the end slope of the tendon is available to act in opposition to the shear force at the end of the beam. If the wires are straight because of manufacturing or other reasons, then it will be necessary to keep the centre of gravity of the tendons within the kern of the section.

This reduction in the shear force is not the only improvement in shearing resistance brought about by prestressing. A prestressed concrete beam is a beam of a homogeneous material under working load—as at this stage the concrete will not have cracked—

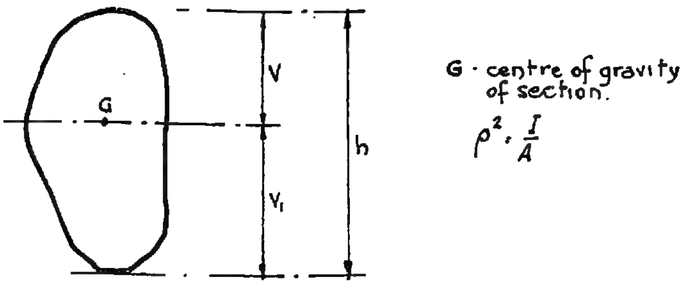


FIGURE 5

FIGURE 6

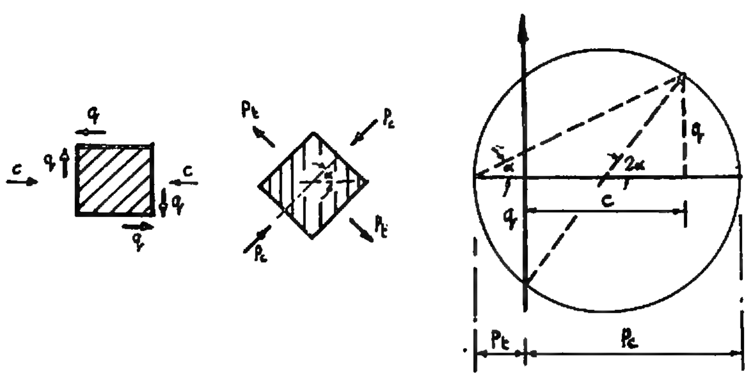


FIGURE 7

consequently we can compute the shearing stress from the well-known formula :—

$$q = \frac{VAy}{bI}$$

This becomes a maximum at the neutral axis, and in the case of a rectangular beam is $1\frac{1}{2}$ times the mean stress. However, the shearing stress is not the main consideration, in concrete, for it is the principal tension which we must limit if the beam is not to fail. In normal reinforced concrete construction the term "principal tension" or "diagonal tension" is not used so frequently because it is numerically equal to the shearing stress where no longitudinal or transverse stresses are present.

The values of the principal stresses are given by

$$p_c = -\frac{C_1+C_2}{2} - \sqrt{\frac{(C_1-C_2)^2}{4} + q^2}$$

principal compression

$$p_t = -\frac{C_1+C_2}{2} + \sqrt{\frac{(C_1-C_2)^2}{4} + q^2}$$

principal tension

where C_1 = longitudinal stress : C_2 = transverse stress :

q = shearing stress at the neutral axis.

The principal stress may very conveniently be obtained graphically by Mohr's circle.

For reinforced concrete the conditions may be seen to be those of pure shear and in this case the principal stresses are at 45° to the shearing stress and equal to them : the form of Mohr's circle being such that the centre of the circle coincides with the origin of the principal stresses, Fig. 6.

In the prestressed beam, conditions are different, for we have a longitudinal stress C_1 which combines with the shearing stress to reduce the principal tension in value and to alter its orientation : this is as shown in Fig. 7.

The graph given in Fig. 8 shows the relationship between C_1 and p_t ; we cannot entirely eliminate p_t with an undirectional prestress, but nevertheless it is considerably reduced.

By introducing a second prestressing force C_2 acting at right angles to C_1—that is, by using stressed web reinforcement—p_t may become zero or a compressive stress. Mohr's circle for this condition is shown in Fig. 9.

Under certain circumstances—such as a beam designed to resist torsional stresses—it may be necessary to introduce a third prestressing force at right angles to the longitudinal and to the vertical web prestressing.

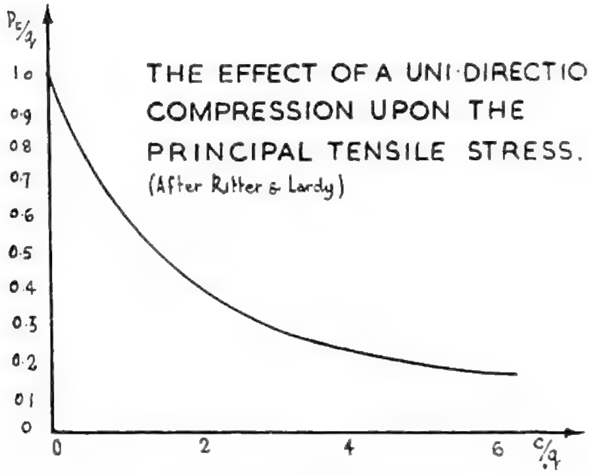


FIGURE 8

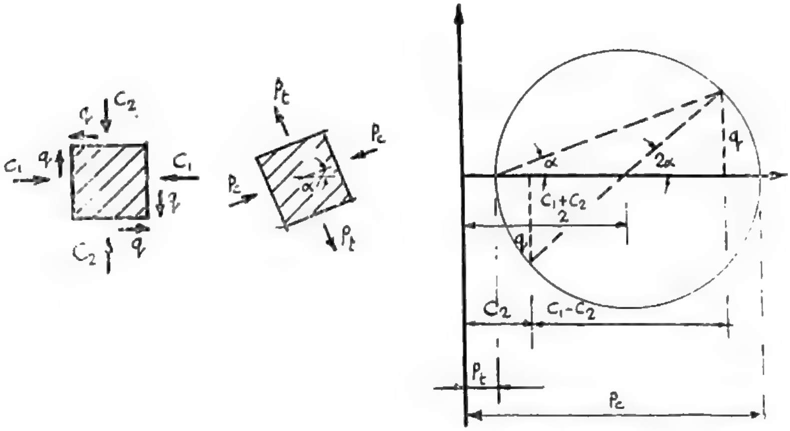


FIGURE 9

Methods of Prestressing and Applications.

The ease with which we may adjust both the magnitude and direction of the force when using single wires or cables formed of a number of wires, leads to their almost universal adoption. There are two main categories into which we may divide the systems using wires or cables :—

(*a*) the pretensioned system.
(*b*) the post-tensioned system.

The Pretensioned System.

In this method the wires are tensioned and held between fixed heads before the concrete is cast round them ; once the concrete has attained a sufficient strength, the wires are released and by reason of their bond with the concrete, put it in a state of compression as their elastic shortening takes place. No permanent forms of anchorage are required. The system is often referred to as the Hoyer system, named after the German engineer Hoyer, who introduced it into Germany in 1938. The originator of the system is rather difficult to determine as Freyssinet claims that his patents of 1928 also cover this method, while recently a plea for the recognition of Wettman as being the first to utilize it has been advanced by Polikiva.

Modifications have been introduced by Abeles and Billig in this country and especially has the method been used here for the production of railway sleepers on a "long-line" process, to a design by the late Dr. K. W. Mautner. The system is particularly suited to the production of flooring units or beams under closely controlled factory conditions in which members of the same section, but not necessarily of the same lengths, are cast in one line and are separated by plates through which the wires pass from a fixed abutment at one end to the straining device at the other. An efficient factory layout can be arranged. The number of jacks required is reduced to a minimum, time is saved in straining all the wires for one line of units at once, and rapid curing of the concrete may be effected by making the dividing walls of the forms between the units of a hollow section with steam pipes running through them. Disadvantages are its high initial capital cost and the difficulty of meeting changes in design which would require changes in factory layout.

M. Freyssinet has introduced a method of making prestressed precast flooring units in a single mould system, the mould being used to take the wire reactions before the unit is demoulded. The concrete is steam cured and is sufficiently strong to resist the prestressing force in two hours.

One of the most important considerations in the pretensioned bonded constructions is to ensure adequate bond resistance. This

is made up of three components : (*a*) an adhesion between the concrete and the steel ; (*b*) a frictional resistance to slipping of the steel by reason of the shrinkage of the concrete, and (*c*) a radial expansion at the free ends of the wire proportional to Poisson's ratio. Tests carried out at the University of Leeds by G. Marshall, Esq., B.Sc. (1)* indicate that for 0·2″ (5 mm.) wire a grip length of 150 diameters is required for a pretension stress of 170,000 p.s.i. ; similarly for 0·08″ (2 mm.) diameter wire a Swiss report (2) gives a required grip length of 170 diameters. Careful concrete placement around the ends of the wires is of great importance in obtaining satisfactory bond. Messrs. Weinberg and Kravtzoff (3) basing their recommendations upon lengthy experience in France, state that with a tension of 170,000 p.s.i. and unit lengths between 9′0″ and 12′0″ the diameter of the wire should not exceed 2·5 mm. and that for shorter lengths a reduced steel stress is advisable (about 140,000 p.s.i.).

Amongst the more interesting structures that have been built using bonded wires, may be mentioned some 9,000 gallons overhead tanks, in use by the French National Railways. These are 18′8″ diameter at the top, 12′4″ at the bottom, with a height of 8′0″ : the shape is that of a hyperboloid (a figure generated by the revolution of inclined straight lines) and thus straight prestressed wires only had to be tensioned. The walls of the tanks were 3 cms. thick and were formed by gunite. Shell roofs of a somewhat similar nature have been advocated by Billig.

Information regarding the application of this technique is available in numerous publications, certain of these being referred to in the bibliography : particularly should be noted the M.O.W. National Building Study Bulletin No. 12—"Plant for Prestressing Concrete."

The Post-tensioned System.

This is probably the more adaptable of the two systems, for it is obvious that there is a limit to the size of precast members that can be cast in a factory and transported, while the construction at the site of forms sufficiently strong to take the wire reactions before release is in many cases hardly a more practicable solution.

The method is to cast the entire member or sections of the member with cavities running through at the correct positions and into which are later introduced the cables, or in some cases, rods. These are then stressed and are anchored at their ends by any one of a number of methods. A variant of this is to cast the member with the cable in position but covered with a sheathing which allows free movement of the wires after the concrete is set.

*References thus (1) indicate that the article mentioned is listed in the bibliography.

In the case of cables, it is advisable, after stressing to grout between the wires along the whole length of the cavity: this may quite easily be done under pressure. The grouting fulfils three functions: it provides a protection against corrosion, an added degree of security against slipping of the cable in the, admittedly remote, possibility of the anchorages failing, and it increases the ultimate strength of the member by ensuring a more uniform distribution of strain between the concrete and the steel.

The losses in the initial wire stress are less with post-tensioning than with pretensioning, as the shrinkage in the concrete, which must occur in setting, has practically all taken place before the tensioning and release of the cables, and furthermore, the losses due to creep may be minimized by casting the members well in advance and ensuring adequate curing.

Two patented forms of construction have achieved great prominence, the Freyssinet method and the Magnel-Blaton method.

That designed by Freyssinet uses a cable of 8, 10 or 12 wires of 0·2″ diameter (5 mm.), (although 14, 16, 18 or 32 wire cables are possible they have not been found necessary in any of the structures yet built by Freyssinet). The wires are arranged around a 16 gauge M.S. wire helix, the whole in certain circumstances being enveloped by a sheet metal sheathing, or a bitumenized covering. The anchorage is particularly ingenious and is the result of much research and long experience on the part of M. Freyssinet. It consists of a precast hollow cylinder of about 3½″ outside diameter and 4″ long, made of a high quality concrete, helically reinforced, the inside cavity tapers from 2″ diameter at one end to 1·1″ at the other. Through this hole the wires are threaded and are anchored in position by means of a shaped conical male cone about 1½″ diameter at the larger end and ¾″ diameter at the smaller. The tensioning procedure is based upon the use of a patented double-acting hydraulic jack. Firstly, the male cones are lightly tapped into place, the jack is then held against the female and the wires are firmly wedged on the rear of the body of the jack. The pump being operated, oil is forced into the tensioning cylinder, thus bringing about the desired elongation of the wires—the elongation is measured directly and checked against the load given by the pressure gauge reading. The latter should not be relied upon by itself. On reaching the required elongation, the oil delivery is switched over to the second piston of the jack, which drives home the male cone, thus forcing the concrete of the male cone in the interstices between the wires with a considerable force, the wires being firmly held upon release of the jack. At the opposite end of the cable the male cone may be driven in with a hammer before tensioning commences.

The cone anchorage is very convenient to use on non-vertical faces, when a precast block containing the female cone may be made

and fixed into the formwork before the mass of the concrete is placed.

The weight of steel in a cone anchorage is about equivalent to 30″ of the cable anchored.

The method of post-tensioning known as the Magnel-Blaton system was devised by Professor Magnel during the German occupation of Belgium while M. Freyssinet's anchorages were impossible to obtain. In the Magnel-Blaton cable, which is rectilinear, the wires are arranged in horizontal rows of 4, with a space of about $\frac{3}{16}$″ between each wire; horizontal and vertical spacers are provided to maintain these clearances.

The anchorage consists of a cast steel distributing plate through which the wires pass to a locking device consisting of a grooved steel plate and wedges. Each groove and wedge anchors two wires. The cable may be metal sheathed and placed in the forms before concreting or it may be threaded through a hole formed by a removable core. Grouting is carried out as described previously. The wires are stretched two at a time by means of a jack. The elongation being measured and the desired length of extension achieved, the wedges in the grooved plate (the "sandwich plate") are driven home by means of a hammer and steel bar.

The advantages claimed for this method by Professor Magnel are :—

(1) The wires being stretched only two at a time may more easily be brought all to the same extension.

(2) The distance between the separate wires composing the cable permits of more efficient grouting.

(3) The jacks may be made lighter and are more easily handled.

Other methods of post-tensioning are available using wires and rods with various anchoring devices. The fact that they are not described here does not indicate that they are unsuitable in design or efficiency. The Freyssinet and Magnel systems have been described in some detail as, up to now, the majority of post-tensioned structures of any size have been built using one or other of these patents.

Methods of stressing not utilizing stretched tendons are due to Freyssinet and Lossier. The former has used an inflatable metal bag, of which one with a diameter of 24″ will exert a force of up to 2,000 tons. It is inflated by means of a cement-water emulsion which will set almost instantaneously upon the reduction of the water content, thus preserving the deformation which brought the force into play. In their right place the use of these "flat jacks" may become extremely valuable—an example is the adjusting of the pressure line of the Beni Badhel Dam in Algeria, the height of which was required to be raised from 60 metres to 67 metres after completion, thus throwing the resultant line of thrust on the

foundation outside the limits of safety. The solution consisted in the insertion of "flat jacks" between the old counterforts and new abutments, the total force exerted, of the order of 500,000 tons, bringing about an even more favourable distribution of pressure beneath the foundations than that which existed before.

M. Henri Lossier has advocated the use of expanding cements which are basically of the Portland type, but with the addition of an expanding agent (a sulpho-aluminate cement) and a stabilising agent. The method of use is to cast a block of concrete made with expanding cement at the point where the thrust is to be applied, for example, at the keystone of an arch or at the top of piles supporting under-pinning beams. Watering holes are left in the blocks at frequent intervals and when the concrete has set, water is introduced into these holes penetrating the mass of the expanding cement. This brings about a swelling of the concrete and when a sufficient expansion has taken place the water is drained away. Within a few hours the expansion stops and the stabilizing agent then begins to absorb the principal cause of expansion, *i.e.*, the calcium sulphate. M. Lossier has already carried out a number of bridge repairs and underpinning works using this technique and making much simpler what would, in the normal way, have been most difficult jobs to undertake.

The construction of large circular reservoirs has been carried out for a number of years in the U.S.A. by binding otherwise unreinforced, or at the most, lightly reinforced, walls by means of rods threaded and tightened with turnbuckles, or by a specially designed machine known as a "merry-go-round" which travels round the circumference supported by a track at the top of the wall and which bind the walls helically.

Prestressing by means of the thermal expansion caused by passing an electric current through a rod and taking up the extension by a nut on the threaded end, is also popular in America. The rod is coated with a plastic material which melts on heating, allowing the free extension of the rod and which sets hard upon cooling, thus restoring the bond.

Finsterwalder and Dischinger have devised systems which are in use in Germany but which are outside the generally accepted scope of prestressed concrete. Both are, in effect, methods of tensioning external ties.

From the foregoing, it may be seen that prestressing techniques can be applied to all kinds of structures, from fence posts to dams, in fact. The ultimate value of any method, however, lies not so much in the range of its application as in its economy compared with the classical methods available for the same purpose, remembering that economy is not to be judged by first costs alone, but that maintenance and other charges may well outweigh these in the finality. It is extremely difficult to generalize about costs,

but it may be said that prestressed structures will compare favourably with normal construction in those cases where

(*a*) spans are large.

(*b*) dead weight is important.

(*c*) there is repetition of small sized members.

In certain cases, the problem itself may indicate prestressing to be the most satisfactory solution. In underpinning, for example, we may introduce small precast elements into the original structure, disturbing it in the minimum. Later by means of cables passed through holes or slots left in the precast units, we can tie the separate elements together to act as a complete beam. Similarly with concrete water retaining structures of any size, prestressed concrete, with its freedom from cracking, its great economy in the use of steel (in normal reinforced concrete the steel stress would be limited to 12,000 p.s.i.) appears to have great possibilities.

Chapter II.

MATERIALS FOR PRESTRESSED CONCRETE.

Concrete.

Progress in prestressed concrete could not have been achieved without a good understanding of the properties of concrete, and it is necessary that the designer should be able to appreciate the limitations of the material in order that he may produce the most economic structure. The effects of shrinkage and creep have already been mentioned and while it may not be correct to say that these two phenomena are of more importance in prestressed concrete than in reinforced concrete, years of experience with the latter have shown, within broad limits, how much importance is to be attached to them. With prestressed concrete this is not so, for our experience is limited and the designer should study the properties of both steel and concrete with the same care that he devotes to the study of stress analysis.

Questions of durability, resistance to weathering and chemical attack should all find their place in the appraisal of what is required from any particular mix of concrete, as well as the question of strength. Although the attainment of maximum possible strength should not be the sole criterion, the results of compressive strength tests are of the greatest value, as there is possibly no other test which is so reliable an indication of other physical properties.

Compressive Strength of Concrete.

Under given conditions with similar materials, the strength in compression of concrete, within the range of workable plasticity, will be determined by the amount of water used. This most important relationship was not generally recognised until the publication in 1918 of the report on "The Design of Concrete Mixes" by Duff Abrams, although it had in fact been stated by both Zeliansky and Feret earlier. Feret expressed it as a function of the ratio of cement to the total volume of voids, which emphasises more fully the importance of thorough compaction of the concrete.

To obtain a high strength concrete we must, then, produce a concrete with the minimum amount of voids. This will be obtained by using a cement that will combine actively with the water to produce a hard solid matrix which will hold together the aggregate, by using the largest size of aggregate that conditions will permit, by a careful grading of the aggregate and by arrangement of the aggregate in the mix so as to obtain the maximum density, that is, by adequate compaction. In any workable mix the amount of water used will be sufficient to ensure hydration of the cement. The actual amount to be added is largely determined by that which is needed to make the mix sufficiently plastic to place and this will be affected by the size and grading of the aggregate—the finer the grading, the more water will be required—and by the amount of cement used.

Low cement contents are advisable to reduce shrinkage and moisture movement, and a low water cement ratio improves the creep properties of concrete.

The low water cement ratios necessary to obtain the strengths required for prestressed concrete work normally require the use of vibration or vibration and pressure to produce adequate compaction. Vibrators are of two types, the surface vibrator, which is attached to or held against the formwork, and the internal vibrator, which is inserted in the concrete. Little is known about the correct amplitude or frequency to be used to obtain optimum results and in many cases vibration is carried out with mechanical hammers held against the form sides. In designing the mix it is important to keep in mind the method of compaction to be used, for it is known that a mix suitable for manual compaction may be quite unsuitable for mechanical compaction.

Mix Proportioning.

The problem of mix design, then, resolves itself into :—

(1) The choice of water cement ratio which experience shows will give the required strength.

(2) The choice of the maximum size of available aggregate suitable for the job.

(3) The determination of the minimum ratio of cement to aggregate that will give the required strength.

(4) The selection of an aggregate grading which will give the required workability for the lowest water content with regard to the method of compaction to be used.

Having thus determined the proportions, a trial mix should be made and the crushing strength determined by cube tests. Adjustments to the trial mix can then be made to determine the most suitable and economic proportions to be used on the job.

Specifications for Concrete Materials.

It is essential to use carefully selected materials, to ensure that the samples submitted for inspection are representative and that their quality is maintained in all deliveries.

Cements should comply with the relevant British Standards, which are :—

Ordinary Portland Cement and Rapid Hardening Portland Cement, ...	B.S. 12
Portland Blast Furnace Cement, ...	B.S. 146
High Alumina Cement, ...	B.S. 915

Aggregates should be of strength at least equal to that of the cement paste which binds them together, they should be inert in water and of a low absorption value. They should be durable, clean and free from adherent coatings and organic impurities. The shape of the aggregate particles should be roughly spherical or cuboidal. Elongated or laminated particles should be excluded. The fine and coarse aggregates should comply with B.S. 882.

Early High Strength Concrete.

With prestressed concrete it is very often important to obtain high compressive strengths as early as possible. These may be aided by the use of special cements and accelerators, as well as by special curing treatment. The special cements used are mostly rapid hardening Portland cement and high alumina cement. For the former the increase in compressive strength compared to normal Portland cement is of the order of 25% at two days and 15% at seven days, while the latter rapidly achieves very high strength, 5,000 p.s.i. at 24 hours being easily obtained.

High alumina cement has disadvantages in that it generates a much greater amount of heat during hardening than does Portland cement, with the attendant danger of cracking due to restrained thermal movement. At the same time, it may lose strength when cured at temperatures over 80°F., thus in large masses its own heat of hydration may be sufficient to bring about not only a reduction in strength but actual disintegration.

Of the accelerators, the most used is calcium chloride in proportions of about 2% of the weight of cement. The strength increase is similar to that obtained by the use of rapid hardening Portland cement.

Curing.

It is well-known that adequate curing of the concrete is essential to the proper hydration of the cement grains. Absence of curing leads to lower strengths, greater shrinkage and low durability. It may usually be best carried out by covering all exposed surfaces of the concrete with matting or fabric that can be kept continually moist for as long as possible.

The rate of strength gain may be greatly increased by the use of steam and hot water curing, and where it is required to re-use formwork quickly, consideration should be given to these methods. For information on the correct temperatures to be used to obtain the desired improvements in strength, see the papers by Saul and Collins noted in the bibliography. Too high a curing temperature and too rapid an increase in temperature reduce the strength materially, but this effect may be modified by the simultaneous application of pressure.

High alumina cements must not be steam or hot water cured.

Tensile Strength of Concrete.

This varies from about 12% to 5% of the strength in compression. It does not increase with age at the same rate as the latter, the ratio between the two tending to decrease with increasing compressive strength.

Steel.

The steel used for prestressed concrete is of a higher quality than that used in reinforced concrete work and is normally a hard drawn wire. It may be a single plain wire, a single indented wire or twin or triple twisted wire. Its ultimate strength varies, depending upon the degree of cold working and the particular properties of the alloy. The range is usually from 80 to 150 tons per square inch. The wire most frequently used has an ultimate strength of about 85 tons per sq. in. with 0·2% proof stress of 75% to 80% of the ultimate stress.

Larger diameter rods are available, these having a lower ultimate strength of about 70 tons per sq. in. and a proof stress of 55 tons per sq. in.

The higher Y.P. stresses of drawn wire are an effect accompanying plastic deformation in the material. Finality has not yet been reached concerning to the exact theory of how this is brought about but one hypothesis is as follows :—

The atoms making up the structure of the steel can be regarded as being arranged in certain crystallographic planes along which gliding or slipping movements occur during plastic deformation—on a highly polished specimen these lines, called Luders lines, may be easily observed. In the neighbourhood of the slip plane the crystals break down into small crystallites (10^{-4} cm. across) which are partially rotated into random orientation by the slip movement, which thus breaks the continuity of crystal plane, further gliding being rendered more difficult.

Creep in the steel is often considered important having regard to the high stresses in use, but consideration will show that this is not so important as might be imagined because of the way in which the steel is employed in prestressed concrete, where upon release of the steel the elastic and non-elastic deformations of the concrete brought about by the elastic shortening of the steel so decrease the initial steel stress that the reduction due to possible creep in the steel can be considered negligible. If we regard creep as being the maintenance of a constant deformation for a reducing load or the increase in deformation for a constant load (the bases upon which most creep tests in steel are carried out), it will be seen that due to the interaction of steel and concrete in prestressed concrete, neither of these conditions obtain and the usual creep test can have little relevance for our conditions. Care should be taken, however, to ensure that the final steel stress—*i.e.*, the working stress used in design—is not above the creep stress limit for the particular steel being used.

The mild steel or high tensile steel secondary reinforcement used should be in accordance with B.S. 785, B.S. 1144 or B.S. 1221, whichever is relevant to the quality or form of steel to be used.

Brief Notes on Construction.

In all forms of concrete construction, supervision and inspection should be of a high standard. In prestressed concrete this is essential, for we are using high stresses and requiring more than usual accuracy and care in fixing cable positions. The engineer or foreman in charge should have considerable experience in normal concrete construction and should be particularly instructed on the differences between reinforced concrete and prestressed concrete work.

Formwork.

Formwork should be stout and well made, adequate to stand up to vibration and so designed as to be available for the maximum number of re-uses. It cannot be over-emphasised that the quality and economy of the finished product depends to a great extent upon the care with which the formwork is designed and made.

Where the appearance of the finished product is important, consideration should be given to lining the formwork with a suitable material and to the detail of the joints between lining sheets. Where joint marks are inevitable they should be accepted and their position chosen so as not to spoil the general appearance of the structure. In cases where these joints cannot be hidden they should be emphasised to form a regular pattern.

Adequate cover should be provided, both to mild steel secondary reinforcement and to the cables themselves. For external work this should not be less than 1½ inches.

The cables or the formers for cable ducts should be held firmly in place by means of specially cast concrete blocks of the correct dimensions, or by mild steel fixing stirrups. In both cases they should be so placed as not to be disturbed by concreting or compaction.

Particular attention should be given to the tensioning of the cables. The required extensions should be indicated by the designer on the working drawings, and although the elongation should be taken as the criterion, the gauge reading on the jack should also be noted to avoid over stretching of the cable should its free overall extension be restricted by friction or by the blocking of the cavity by leakage of the concrete through the protective sheathing.

Finally, a word about safety precautions. Wherever wires are being stretched before the concrete is placed around them, it is desirable that adequate guards be provided in case a defective wire snaps. Similarly, with wedge anchor devices both as permanent anchorages and as a means of attaching the wires temporarily to the jacks, care should be taken to prevent injury to workmen brought about by the wedges slipping and flying off.

Chapter III.

DESIGN IN PRESTRESSED CONCRETE.

Stresses.

Within the range of working loads the consideration of maximum allowable stresses for prestressed concrete differs from that for reinforced concrete, because of the fundamental difference in the way in which the steel is used. In prestressed concrete the highest stress in the steel occurs at the time of stressing. Afterwards and up to the transformation load, *i.e.*, the load at which there ceases to be any prestressing forces acting in the member, there is a

decrease in the steel stress of an amount up to 15% of the initial stress. The extensions of the beam following the application of the working load, alters the steel stress by less than 5%. Similarly with the concrete, the maximum stress occurs immediately upon release and is not exceeded within the designed load capacity of the member. With reinforced concrete the stresses increase linearly with the loads.

It has been seen that it is necessary to use high steel stresses to overcome the loss of prestressing force by creep and shrinkage. The steel stresses are usually taken as 80% of the 0·2% proof stress or about 60% to 70% of the ultimate strength of the wire. These proportions, which are higher than those normally adopted, are satisfactory because each wire or cable is automatically tested by the prestressing process to about 10% in excess of the stress to which it will be subjected within the range of working loads.

For the concrete, which is made under site conditions that cannot compare with the careful manufacture of steel, more liberal factors of safety must be adopted. Under full load the compressive stress in bending should not exceed one third of the 28-day ultimate crushing strength of a 6″ cube. At release and for the compressive stress which will be modified by creep, we may take a less conservative view, in this case 40% of the crushing strength of a 6″ cube of the age of the concrete to be stressed.

Tensile stresses in the concrete due to bending, may be allowable, depending upon the type of structure; under full load a slight tension in the bottom fibres of a road bridge beam is not objectionable. In a liquid retaining structure, tensile stresses should be avoided altogether and for beams made up of separate blocks tied together by the cables, compression should exist over the whole cross section. There should not, normally, be any objection to small tensions at the time of release which will disappear in a few weeks as creep reduces the prestressing force. Where permitted the tensile stress due to bending may be taken as not exceeding one half the modulus of rupture.

The principal tensile stress should not exceed one half the modulus of rupture except where suitable reinforcement is provided.

In order to minimise the losses due to creep, etc., it is advisable to use a concrete with a 28 day, 6″ cube crushing strength of not less than 6,000 p.s.i.: at the time of release the crushing strength should not be less than 4,000 p.s.i. With these stresses in the concrete and a steel stress of about 60 tons per square inch, the loss due to all causes may be taken as having a maximum value of 15% of the initial stress for post tensioned work. For pre-tensioned construction, this may have to be increased, reliable test data being used to estimate its value wherever possible.

Notation.

f^1t = stress in top fibres of section at release.
ft = stress in top fibres of section under working load.
f^1b = stress in bottom fibres of section at release.
fb = stress in bottom fibres of section under working load.

In all cases the sign of the stress is indicated as :—

— for compression.
+ for tension.

F_i = initial prestressing force.
F = final value of prestressing force.
η = F/F_i = ratio of final to initial prestressing forces.
A = cross-sectional area of member.
e = eccentricity of cable.
h = height of member.
b = breadth of member.
v = distance from upper fibre to centroid of section.
v_1 = distance from lower fibre to centroid of section.
M_{DL} = bending moment due to loading at time of prestressing (normally that due to self-weight of member).
M_{TL} = bending moment due to total loading on member.
Z = modulus of section with respect to the upper fibre.
Z_1 = modulus of section with respect to the lower fibre.
I = moment of inertia.
ρ^2 = I/A = square of the radius of gyration.
c = ρ^2/v_1 = distance from centroid to upper kern limit.
c_1 = ρ^2/v = distance from centroid to lower kern limit.

Beams Bending in One Direction Only.

The conditions of stress to be satisfied are :—

I. Under the initial values of prestress and self-weight.

Bottom Fibre :

$$\text{(i)} -\frac{F_i}{A}\left(1+\frac{ev_1}{\rho^2}\right) + \frac{M_{DL}}{Z_1} = f^1b \leq$$

allowable compressive stress at release.

Top Fibre :

$$\text{(ii)} -\frac{F_i}{A}\left(1-\frac{ev}{\rho^2}\right) - \frac{M_{DL}}{Z} = f^1t \leq$$

allowable tensile stress at release.

II. Under final values of prestress and total loading.

Bottom Fibre :

$$\text{(iii)} -\eta \frac{F_i}{A}\left(1+\frac{ev_1}{\rho^2}\right) + \frac{M_{TL}}{Z_1} = fb \leqslant$$

allowable tensile stress under working load.

Top Fibre :

$$\text{(iv)} -\eta \frac{F_i}{A}\left(1-\frac{ev}{\rho^2}\right) - \frac{M_{TL}}{Z} = ft \leqslant$$

allowable compressive stress under working load.

From equations (i) and (iii) we may obtain

$$\frac{M_{TL}-\eta\, M_{DL}}{Z_1} = fb-\eta\, f^1b \tag{v}$$

$$Z_1 = \frac{M_{TL}-\eta\, M_{DL}}{fb-\eta f^1b} \tag{vi}$$

and similarly from (ii) and (iv)

$$\frac{M_{TL}-\eta\, M_{DL}}{Z} = \eta\, f^1t-ft \tag{vii}$$

$$Z = \frac{M_{TL}-\eta\, M_{DL}}{\eta\, f^1t-ft} \tag{viii}$$

$$\frac{Z_1}{Z} = \frac{\eta\, f^1t-ft}{fb-\eta\, f^1b} \tag{ix}$$

If the beam is symmetrical

$$f = \eta\, f^1t-ft \leqslant fb-\eta\, f^1b$$

and

$$Z = Z_1 = \frac{M_{TL}-\eta\, M_{DL}}{f} \tag{x}$$

In equations (v) to (x) the values for f^1t, ft, f^1b and fb are to be taken as the permissible stresses with their correct signs.

The problem of stress analysis—that is, checking the stresses for a given section with a known prestressing force and under a known moment, is simple. For any particular position along the span it is only necessary to apply the first four equations.

A more difficult problem is that of dimensioning the section for given moments so as not to exceed the permissible stresses. From the foregoing it may be seen that values for Z or Z_1 may be obtained fairly simply; if the section is to be rectangular then the sizes follow from the well-known formula $Z = bh^2/6$. However, a rectangular section is not economic in the use of the material that goes to make up the section, and I or a T section is to be preferred. In order to eliminate some of the guesswork that would otherwise be necessary to determine the dimensions of a section of this shape, tables have been prepared which will guide the user more quickly to the most economic proportions. The tables are to be found in the appendix, their use is obvious and their basis is that the geometric properties of the section may be determined in terms of the enclosing rectangle.

The author would like to acknowledge that the form of equations (v) to (x) was first given by Dr. P. W. Abeles, in "Concrete and Constructional Engineering" for April, 1948. In the same article Dr. Abeles also suggested the possibility of constructing tables such as those given here.

Having satisfied the minimum requirements for Z and Z_1, the values of F and e may be obtained by considering the stresses required to be induced by prestressing force to overcome the effects of self-weight and applied loading.

If σ_p and $\sigma^1{}_p$ are the stresses at the top and bottom fibres respectively due to the final value of the prestressing force when all relaxations have taken place, it is obvious that

$$\sigma^1{}_p = fb - \frac{M_{TL}}{Z_1} \qquad \text{from equation (iii)}$$

$$\sigma_p = \eta\left(\frac{M_{DL}}{Z} + f^1t\right) \qquad \text{from equation (ii)}$$

then

$$-\frac{F}{A}\left(1 - \frac{ve}{\rho^2}\right) = \sigma_p \qquad (a)$$

$$-\frac{F}{A}\left(1 + \frac{v_1e}{\rho^2}\right) = \sigma^1{}_p \qquad (b)$$

Subtracting (b) from (a) will give

$$e = \frac{(\sigma_p - \sigma^1{}_p)\ I}{Fh} \qquad (c)$$

Substituting for e in (a)

$$-\frac{F}{A}\left(1-\frac{ve}{\rho^2}\right)=\sigma_p \qquad (d)$$

$$-\frac{F}{A}+\frac{v\,(\sigma_p-\sigma^1{}_p)}{h}=\sigma_p$$

$$F=-(\sigma_p\,v_1+\sigma^1{}_p v)\frac{A}{h} \qquad \text{(xi)}$$

and in (c) the value of F from (xi)

$$e=\frac{(\sigma_p-\sigma^1{}_p)\,\rho^2}{(\sigma_p v_1+\sigma^1{}_p v)} \qquad \text{(xii)}$$

For certain stress conditions, zero tension at release and zero tension under working load, it is possible to obtain F and e more simply from

$$F=\frac{M_{TL}-\eta\,M_{DL}}{c+c_1} \qquad \text{(xiii)}$$

$$F_i=\frac{F}{\eta} \qquad \text{(xiv)}$$

and

$$e=c_1+\frac{M_{DL}}{F_i} \qquad \text{(xv)}$$

Throughout the foregoing it is assumed that the section can be dimensioned from the bending moment due to the loads applied after prestressing, plus only that small portion of M_{DL} which the relaxation in F necessitates being added. This is satisfactory up to the limit of M_{DL}/M_{SL} indicated approximately for any section by the expression given on page 9. Where M_{DL} is a greater proportion of the total amount than the section can carry by displacing F then we must increase Z to a value given approximately by

$$Z=\frac{M_{TL}}{ft\left(1+\dfrac{M_{DL}}{\propto M_{AL}}\right)} \qquad \text{(xvi)}$$

in which $\propto \;=\; \dfrac{e_{max}}{v_1}$: e_{max} allowing for cover to cables, etc.

$\propto$ may frequently be taken as 0·8.

M_{AL} = moment due to load applied after prestressing.

Whether or not it is necessary in computing the properties of the chosen section to take into account the areas of the holes

required for the cables, depends upon their area in relation to that of the cross section. Similarly, with pretensioned or post-tensioned, fully grouted beams, the effect of the transformed area of steel may be included if it is considered of sufficient importance, but as the steel area is generally small this would appear to be an unnecessary refinement. If, however, it is included it can only be effective in resisting stresses due to the applied load ; the concrete alone must resist the effects of the prestressing force.

The value of m may be taken as 5 for the effects of loading applied immediately upon release or for loads of short duration during the life of the structure and as 20 for loads of long duration.

A prestressed beam acts quite differently from a reinforced concrete beam and it is important to appreciate this difference fully. In an r.c. beam the reinforcement plays a static role in resisting stress but in a prestressed beam the "reinforcement" (*i.e.*, the cable), itself causes stresses ; at the section of maximum moment these stresses are opposed by those brought about by the external forces but it is obvious that if the cable eccentricity is constant throughout the length of the beam, then at points remote from that of maximum moment stresses which the beam cannot resist will be set up. It is thus necessary that the eccentricity of the cables be reduced proportionally to the falling off of the dead load moment and in such a manner that at any point the eccentricity is sufficient to overcome the effects of the live load moment. This may be done by continuous application of the formula given above, but this is a time consuming business and a graphical method due to Guyon is available, which is of much greater value.

If we consider the internal action of the prestressing forces in the beam under load, we see (Fig. 10) that at the support the reaction R and the prestressing force F create a resultant C ; as we get away from the support both the inclination of F and the value of the shearing force are reduced until at the centre line F and C are horizontal. At this point, it may be seen that the internal moment, which must equal the external moment is equal to $F \times a$ and obviously $a = M/F$.

Let us consider the cross-section of the beam, Fig. 11, on which are marked the limits of the kern section C and C_1. Under the influence of the dead load the centre of pressure rises by an amount M_{DL}/F_i, to fall within the kern for there to be no tension on the section, that is to say, the eccentricity of the cable must not lie below B_1 where $C_1 B_1 = M_{DL}/F_i$. Similarly, when live and dead loads act, the centre of pressure must not rise above the upper kern limit, or the eccentricity of the cable must not lie above B where

$$CB = \frac{M_{DL} + M_{LL}}{F}$$

If we draw an elevation of the beam to a suitable scale, marking on it the upper and lower traces of the kern limits, we may use these two lines as bases for, respectively, the curve M_{TL}/F and M_{DL}/F_i. The cable eccentricity should then lie between these two curves for there to be no tension on the section, Fig. 12. Where small tension stresses are allowed in the design the curves will overlap, the cable should then follow the lower curve until the tension zone is passed. See example I, page 44.

It will be seen that where the optimum section is used, and no tensions are allowed, the curves touch at the point of maximum moment. In drawing this diagram, it is advisable to use an exaggerated scale for the depth of the beam.

Members Subject to Reversal of Moment.

Frequently it is necessary to design members to resist a reversal of moment. Obvious examples are precast beams which may be lifted the wrong way up and overhead transmission line poles which may be loaded in any direction.

Considering Fig. 11, it can be seen that the maximum use of any section is made when the addition of the applied load moment causes the centre of pressure to rise from the lower to the upper kern limit, that is to say, the variation in moment determines the section.

Now if we call the maximum positive moment M_1 and the maximum negative moment M_2, the variation in moment is

$$\triangle M = M_1 + M_2$$

We have already seen, from (vi) and (viii) that the required section modulus is given for either face by dividing the moment by the permitted variation in stress on the face. We see then that

$$Z = \frac{\triangle M}{\triangle f}$$

More formally this is expressed

$$Z_1 = \frac{M_1 + M_2}{fb - ft} \geq \frac{M_1 + \eta M_s}{fb - \eta f^1 b} \qquad \text{(xvii)}$$

$$Z = \frac{M_1 + M_2}{fb - ft} \geq \frac{M_2 + \eta M_s}{fb - \eta f^1 b} \qquad \text{(xviii)}$$

It will be noted that in this case the dead weight of the beam cannot be resisted by displacing the centre of action of the prestressing force for we must confine its position within the kern.

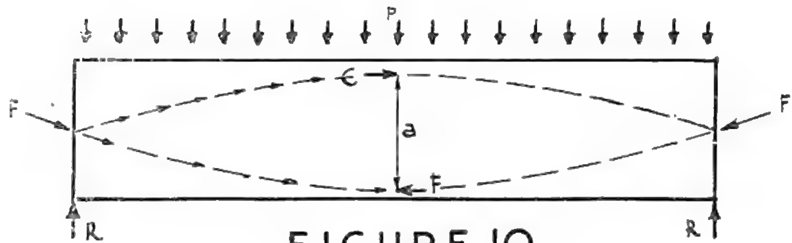


FIGURE 10

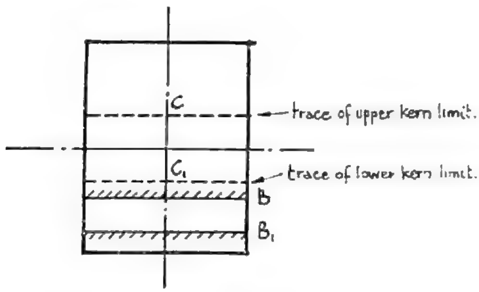


FIGURE 11

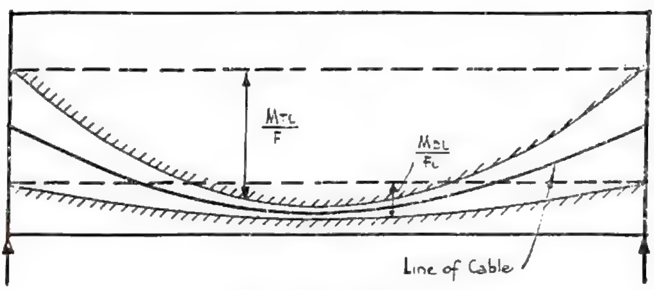


FIGURE 12

In a manner similar to that previously described, we may compute F and e.

$$\text{If} \quad \sigma^1{}_p = fb - \frac{M_1}{Z_1} < \frac{M_2}{Z_1} + ft$$

$$\text{and} \quad \sigma_p = \frac{M_1}{Z} + ft < fb - \frac{M_2}{Z}$$

$$F = -(\sigma_p v_1 + \sigma^1{}_p v) \; A/h$$

$$\text{and} \quad e = \frac{(\sigma_p - \sigma^1{}_p) \; \rho^2}{(\sigma_p v_1 + \sigma^1{}_p v)}$$

Shear.

We now have to consider the effects of shearing force on the stresses in the beam. As has been discussed previously, prestressing reduces the value of the principal tension by reason of the compressive stresses set up within the beam and, where the cables are curved upwards at the supports, as is generally the case, the actual value of the shearing force is reduced by the vertical component of the force in the cable, Fig. 13. This vertical component has a value of F sin α and if the cable lies in a parabolic curve it may easily be seen that the vertical forces due to the cable vary linearly from T at the supports to zero where the cable becomes horizontal, T having the maximum value of

$$T = \frac{F}{\sqrt{\left(\frac{l}{4r}\right)^2 + 1}}$$

where r = the rise of the cable from its lowest point to the anchorage.

The method of combining the shearing stresses, the longitudinal stresses and, in some instances, the transverse stresses to give the principal stresses is explained on page 11.

Where there are many cables in a girder it may appear advantageous to bend the cables up from the bottom flange to the top flange and anchor them there, in the manner of diagonal bars in reinforced concrete. Indeed this method has been adopted, notably in the bridge by Freyssinet at Luzancy. Care has to be exercised in the analysis to ensure that there is a sufficient force in the remaining cables to meet the requirements for bending, and a suitable method is given by Guyon in ref. 4 ; Barat, in ref. 5, has devised a rapid method of selecting the optimum spacing for the bending up of the cables. The disadvantages of this system lie in the practical difficulties encountered in tensioning cables bent through relatively sharp angles and in arranging suitable positions for the anchorages. Fig. 14 shows methods of placing the cables ;

(*a*) is more suitable than (*b*) because of the reduced friction losses, but if (*b*) is adopted then two jacks should be used simultaneously. The radius of curvature should be as great as possible and not less than 20′ in any case.

Where the principal tension exceeds that allowable, it is necessary to take all the stress on stirrups of mild steel or high tensile steel. The area and spacing of the stirrups can be determined from the values of the principal tension.

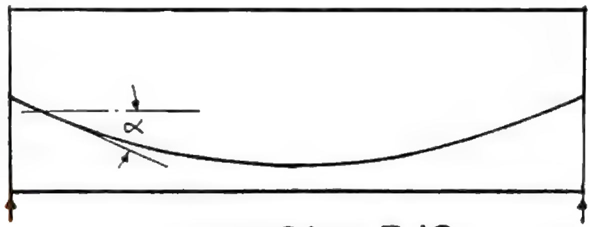


FIGURE 13

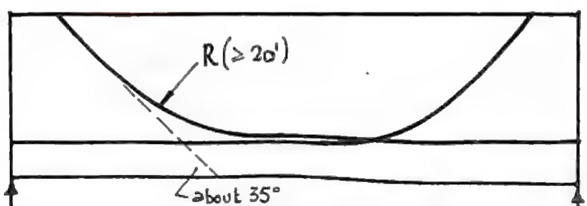


FIGURE 14 A

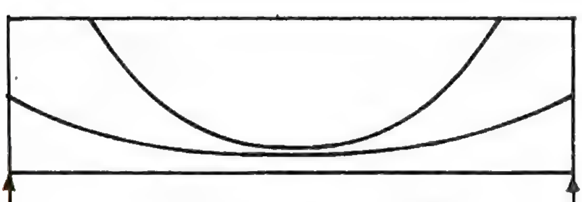


FIGURE 14B

Stresses at Anchorages.

Other than in zones close to the anchorages, the distribution of stresses on the section, due to the prestressing force, is in accordance with the principle of St. Venant, that is to say, it depends only on the position and magnitude of the resultant. Close to the anchorages this is not so and very high local stresses may be set up.

At the moment there is no established method of computing these stresses and the design of the anchor blocks must be based mainly on the engineering judgment of the designer. Magnel in his book *Prestressed Concrete*, gives a method of analysis based upon a consideration of the end block as a beam supported by the anchorages on one face and loaded on the opposite face by reactions from the adjacent beam section. Although certain authorities consider this method to overestimate the principal tensions, tests appear to indicate that it may be used with confidence.

To reduce the undesirable high tensions which can occur, it is necessary to distribute the anchorages over as much of the end section as possible so that there will be little tendency for a high shearing stress to develop. Where the beam is of an I or T cross-section, it is often advisable for the end blocks to be rectangular in section for a length about equal to their height. Where this is done it is unlikely that tensions will occur which cannot be resisted by the concrete. If reinforcement is necessary it should be of the form of mild steel grills of about $\frac{1}{4}''$ diameter, the amount required being based on the value and direction of the principal tension.

In all cases one or two grills should be placed behind the anchorages to help distribute the high compressive stress and where the anchorages are embedded in the concrete they should be surrounded by helices of mild steel rods of the same diameter.

The permissible stress for localized loading such as that under the anchor plate may be increased beyond the stress allowable for a load distributed over the whole cross-section.

If f_1 is the increased stress under localised loading.
f is the generally permitted stress on the cross-section.
A is the total area of cross-section.
A_1 is the area of distribution of f_1.

$$\text{then } f_1 = f\,\sqrt[3]{A/A_1}$$

The centroids of A and A_1 should coincide.

The stress f_1 may also be increased by the use of helical or other binding in the area A_1. This increase may be taken as the product of the permissible stress f_1 computed from the foregoing formula times the following coefficient

$$1 + \mu w \left(1 - \frac{2p}{a}\right) \frac{fs}{f_1}$$

in which

μ is a coefficient depending upon the form of the reinforcement and has the value

5·6 for a continuous helical binding or for two layers of rods forming a square mesh, the rods in each direction being continous or in the form of square welded mesh.

2·8 for square stirrups.

2·8 a/b for rectangular stirrups, a being the smaller and b the larger dimension of the stirrup.

w is the volume of binding per unit volume of concrete.

p is the pitch of helices or the spacing of each "mat" of square mesh.

a is the least dimension of the compressed section.

f_s is the steel stress at its elastic limit.

In no case shall the permitted increase in f_1 be greater than 2·5.

Deflection.

Prestressed concrete beams exhibit less deflection than do comparable reinforced concrete beams under the same load. This is principally due to the initial upward camber brought about by the prestressing force, but some smaller beneficial effects are to be noticed. Amongst these is the elimination of tensile cracks under working load conditions and consequent increase in the value of I.

During the loading of prestressed beams to their ultimate capacity, they pass through three definite stages. In the first, the tension in the tendons is practically independent of the loading of the beam. At the limit of this stage, which is known as the transformation load, the increase in load will be accompanied by an increase in steel stress and, more important, the concrete in the tensile zones will be cracked. There will be a marked increase in deflection at the transformation load, which is brought about by the reduction in the effective concrete area. Nevertheless, upon removal of the excess load, the cracks will close up and the beam will act as before overloading. A second limit marks the beginning of the third stage. At this point the load brings about an irreversible strain in either the concrete or the steel and continued loading rapidly causes failure.

Continuous Structures.

It has been seen that in simply supported structures the pressure line for the prestressing force is coincident with the axis of the cables when there are no loads acting on the member, and that under the influence of external loads the pressure line rises by an amount M/F ; with continuous structures this is not so, for the prestressing force, itself tending to deflect the member, brings into play additional forces at the supports which, in turn, cause deviations

of the pressure line from the axis of the cable before external loads are applied, thus setting up further moments in the member.

An example will make this clear. The shape of the cable is chosen purely to demonstrate the method and does not represent a typical continuous prestressed beam ; its form was chosen to simplify the calculations.

Consider a beam ABC, Fig. 15, of two equal spans, prestressed with a force F, the line of the cable being parabolic with a maximum eccentricity e at support B.

At any point x the eccentricity is $ex\,(2l-x)/l^2$ and the member has a moment induced in it

$$M_x = \frac{Fex\,(2l-x)}{l^2}$$

assuming that α the angle of slope of the cable is small enough for $\cos\alpha$ to be taken as unity.

If we remove the centre support,

$$\delta_B = -\frac{Fe}{EI\,l^2}\int_0^l x^2\,(2l-x)\,dx$$

$$= -\frac{5\,Fel^2}{12\,E\,I}$$

To maintain the original position of B we will have to apply a force P sufficient to bring about a deflection equal and opposite to that caused by the prestressing force.

$$-\frac{P\,(2l)^3}{48\,EI} = -\frac{5\,Fe\,l^2}{12\,EI}$$

$$P = \frac{5}{2}\;\frac{Fe}{l}$$

The force P causes a moment of opposite sign to that brought about by the prestressing force, its value being

$$M_P = \frac{5}{4}\,Fe$$

Thus the final moment diagram due to prestressing alone will be, referring to Fig. 15, the sum of diagrams (b) and (e).

From the foregoing it may be seen that it is fairly simple to determine the support moments due to prestressing for any assumed line of cable and value of F. For each span there may be set up equations based upon the principles of moment areas, similar to the three moment equations. Thus from Fig. 16, we have, taking the support M_o :—

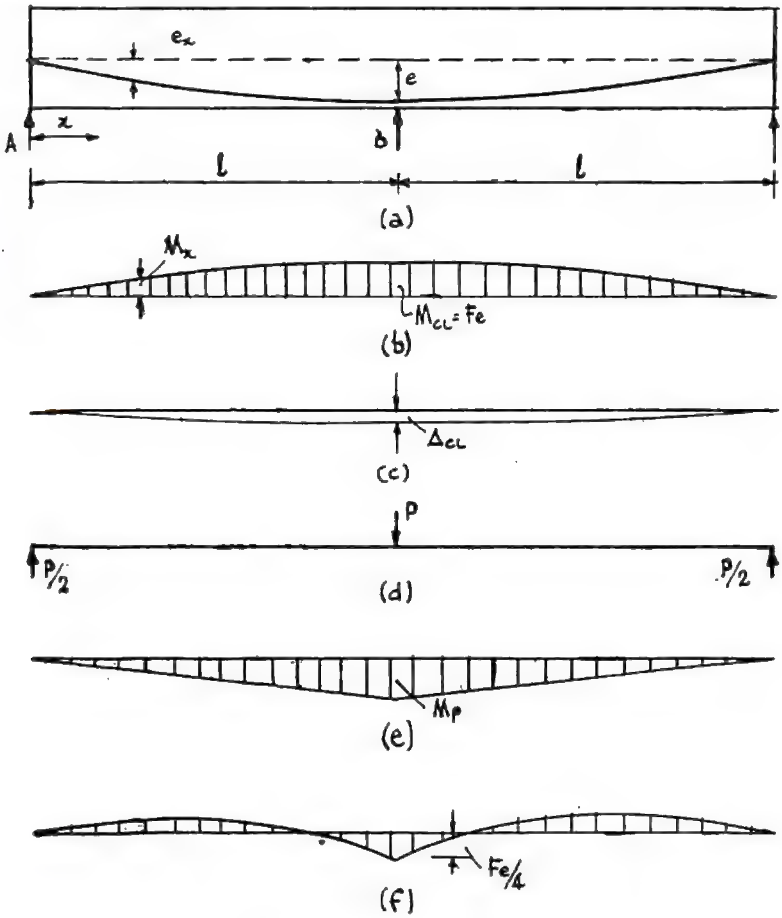


FIGURE 15

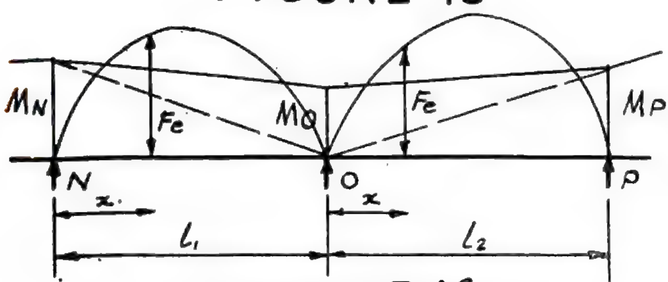


FIGURE 16

$$M_N \int \frac{x\,(l_1-x)\,dx}{I\,l_1^2} + M_o \int \frac{x^2\,dx}{I\,l_1^2} = F \int \frac{e\,x\,dx}{I\,l_1}$$

$$M_o \int \frac{(l_2-x)^2\,dx}{l_2^2\,I} + M_P \int \frac{x\,(l_2-x)\,dx}{I\,l_2^2} = F \int \frac{e\,(l_2-x)\,dx}{I\,l_2}$$

Similar equations may be set up at each joint, evaluated by numerical integration methods and solved simultaneously for the support moments M_N, M_o, etc.

The choice of the curve for the cable presents the difficulty in design. Broadly speaking, there are two ways open to us ; we may move the cable up and down in a beam of constant section or we may keep the cable as straight as possible, altering the eccentricity by changing the concrete section. Due to the difficulty of construction and the frictional losses involved in the first method, the second is to be preferred.

The paper by Roberts & Lebelle, ref. (6), describing the construction of the first of the new concrete reservoirs at Orleans should be studied by all who are interested in the design of continuous prestressed structures and also of real value is the paper by M. Y. Guyon, ref. (7).

Amongst others, two points require care in detailing; one is that there shall be no sharp changes of direction in the cables, as we shall both lose a large proportion of the prestressing force in overcoming friction and set up high local stresses at the change in direction. Also to be watched is that there is not a large reduction in the load factor, for under possible overload, as we shall see later, the beam tends to act similarly to a normal r.c. beam and with a continuous prestressed beam there may not be any steel near the tension face.

Ultimate Loads.

The ultimate load that may be carried by a prestressed concrete beam depends to a great extent upon the method adopted for prestressing. For pretensioned beams and for post-tensioned beams that have been effectively grouted so that complete bond with the concrete of the beam may be assumed, the computation is similar to that proposed by Whitney for r.c. beams, for the prestressed beam at this stage is acting as an r.c. beam. In most cases it will be seen that because of the high steel stresses employed, the steel percentage will be low and failure will be due primarily to the yielding of the steel. Many engineers consider that "over-reinforced" prestressed beams—that is, those in which failure will be due to the concretes crushing before the steel has reached its yield point, should be avoided,as failure occurs suddenly and without any warning.

Beams in which the cables are not grouted effectively to bond with the concrete, have much reduced ultimate strengths compared to bonded beams. There is a greater tendency for sudden failure, and it is noticeable in testing these beams that this occurs by the formation and widening of one crack at a point of maximum stress. This is brought about by the steel being free to stretch over its entire length, considerable elongation in the concrete causing only a slight increase in steel stress which does not generally reach its yield point, whereas in the bonded construction the steel deformation must follow that of the concrete. It may be advisable to regard the ultimate load for a non-bonded beam, as suggested by Professor A. L. L. Baker, as being the load at which tensile cracks first appear in the concrete.

For bonded, under-reinforced beams, the ultimate moment may be deduced by assuming a rectangular stress distribution for the concrete at failure with a maximum value equal to 0·85 times the cube crushing strength. The total tensile force in the wires is the product of their area and the yield point stress of the steel, and as the total tensile force must equal the total compressive force a value for the depth of the neutral axis may be deduced. The lever arm, a, will then be the distance from the centre of the steel to the centre of action of the compressive force and the ultimate moment will be $Ta = Ca$.

If stress-strain curves for the concrete and steel are available an excellent semi-graphical method is described by Professor Dr. Ing. Mörsch in ref. (8). This is based upon an hypothesis of plane strain distribution, the validity of which was confirmed for the author by the Stuttgart Laboratory for Testing Materials. By the assumption of maximum strain in either steel or concrete, it is possible to determine various positions for the neutral axis and from these positions and with the aid of the stress strain curves to compute values for T and C. These latter may then be plotted graphically against the neutral axis depths from which they were calculated, and both T and C being plotted on the same graph, their points of intersection determine that position of the neutral axis for which T equals C. The lever arm may then be determined and the moment follows.

Numerous theories for assessing the ultimate strength of reinforced concrete and prestressed concrete beams have been published and there is a considerable amount of research being carried on at the moment. Further information may be obtained by reference to the papers of Evans, Baker and Guerrin (refs. 9, 10 and 11).

Chapter IV.

EXAMPLES OF PRESTRESSED CONCRETE DESIGN.

Example 1.

Design a rectangular beam to be cast *in situ* and post tensioned to carry a precast unit floor on a span of 30′ 0″. The loading is 1000 lbs./ft. due to the dead weight of the floor units and 1000 lbs./ft. due to superimposed load on the floor.

Allowable Stresses.

Allowable compressive stress at release of cables	=	1750 p.s.i. (f^1b).
Allowable compressive stress at working load	=	1500 p.s.i. (ft).
Allowable tensile stress at release of cables	=	150 p.s.i. (f^1t).
Allowable tensile stress at working load	=	0 (fb).
Principal tensile stress	=	125 p.s.i. (pt).
Tensile stress in wires at working load	=	120,000 p.s.i. (t).
η	=	0·85.

Moments Due to Applied Loading.

$M_1 = 1{\cdot}0 \,.\, 30^2 \,.\, 1{\cdot}5 \quad = \quad 1350$ Kip ins. due to floor dead load.
$M_2 = 1{\cdot}0 \,.\, 30^2 \,.\, 1{\cdot}5 \quad = \quad 1350$ Kip ins. due to superimposed load.

Total applied load moment 2700 Kip ins.

(*Note.*—1 Kip = 1000 lbs. weight).

Compressive Stress to be Used in Design.

$$f = \eta f^1t - ft \leqq fb - \eta f^1b$$
$$f = 0{\cdot}85 \,.\, 150 + 1500 \leqq 0 + 0{\cdot}85 \,.\, 1750$$
$$= 1490 \text{ p.s.i.}$$

Assume moment due to self-weight of beam $= M_{DL} = 500$ Kip ins. then $M_{TL} = 2700 + 500 = 3200$ Kip ins.

$$Z = \frac{3200 - 0{\cdot}85 \,.\, 500}{1490} = \mathbf{1865} \text{ ins.}^3$$

Let $b = h/3$, then

$$\frac{h^3}{18} = 1865$$

$$h = 32{\cdot}3''$$
$$\text{say } h = 32''$$
$$b = 11''$$

Properties of Chosen Section.

$A = 352$ ins.^{2} $\qquad \rho^2 = 85{\cdot}6$ in.^{2}

$I = 30100$ ins.^{4} $\qquad v = v_1 = 16{\cdot}0$ ins.

$Z = Z_1 = 1880$ ins.^{3} $\qquad \rho^2/v = \rho^2/v_1 = 5{\cdot}33$ ins.

Actual $M_{DL} = 495$ Kip ins.

Final Values of Stresses Required to be Induced by Prestressing.

$$\sigma^1 p = fb - \frac{M_{TL}}{Z_1}$$

$$= 0 - \frac{3195}{1880} = -1700 \text{ p.s.i.}$$

$$\sigma p = \eta \frac{M_{DL}}{Z} + \eta f^1 t$$

$$= 0{\cdot}85 \,.\, \frac{495}{1880} + 0{\cdot}85 \,.\, 150 = 351 \text{ p.s.i.}$$

Required Prestressing Force.

$$F = -(\sigma_p v_1 + \sigma^1{}_p v)\, A/h$$

$$= -(351 \,.\, 16 - 1700 \,.\, 16)\, \frac{352}{32}$$

$$= -\textbf{237 Kips.}$$

Initial Value of Prestressing Force.

$$F_i = F/\eta$$

$$= 237/0{\cdot}85 = \textbf{279 Kips.}$$

Eccentricity of Prestressing Force.

$$= \frac{(\sigma_p - \sigma^1{}_p)\, \rho^2}{(\sigma_p v_1 + \sigma^1{}_p v)}$$

$$= \frac{(351 + 1700)\ 85{\cdot}6}{(351 \,.\, 16 - 1700 \,.\, 16)} = \mathbf{8{\cdot}13''}$$

Check on Stresses due to Bending and Prestress.

At release of cables :—

Bottom fibre.

$$-\frac{F_i}{A}\left(1 + \frac{ev_1}{\rho^2}\right) + \frac{M_{DL}}{Z_1} = f^1 b$$

$$-\frac{279}{352}\left(1+\frac{8\cdot13\,.\,16}{85\cdot6}\right)+\frac{495}{1880}=-2000+263$$

$$=-1737 \text{ p.s.i. (compression)}$$

Top fibre :

$$-\frac{F_i}{A}\left(1-\frac{ev}{\rho^2}\right)-\frac{M_{DL}}{Z}=f^1t$$

$$-\frac{279}{352}\left(1-\frac{8\cdot13\,.\,16}{85\cdot6}\right)-\frac{495}{1880}=+413-263$$

$$=+150 \text{ p.s.i. (tension)}$$

Under reduced prestress and dead weight of floor.

Bottom fibre :—

$$-\frac{F}{A}\left(1+\frac{ev_1}{\rho^2}\right)+\frac{M_{DL}+M_1}{Z_1}=fb$$

$$-\frac{237}{352}\left(1+\frac{8\cdot13\,.\,16}{85\cdot6}\right)+\frac{1845}{1880}=-1700+983$$

$$=-717 \text{ p.s.i. (compression)}$$

Top fibre :

$$-\frac{F}{A}\left(1-\frac{ev}{\rho^2}\right)-\frac{M_{DL}+M_1}{Z}=ft$$

$$-\frac{237}{352}\left(1-\frac{8\cdot13\,.\,16}{85\cdot6}\right)-\frac{1845}{1880}=+350-983$$

$$=-633 \text{ p.s.i. (compression)}$$

Under reduced prestress and full load.

Bottom fibre :—

$$-\frac{F}{A}\left(1+\frac{ev_1}{\rho^2}\right)+\frac{M_{TL}}{Z_1}=fb$$

$$-\frac{237}{352}\left(1+\frac{8\cdot13\,.\,16}{85\cdot6}\right)+\frac{3195}{1880}=-1700+1700=0$$

Top fibre :

$$-\frac{F}{A}\left(1-\frac{ev}{\rho^2}\right)-\frac{M_{TL}}{Z}=ft$$

$$-\frac{237}{352}\left(1-\frac{8\cdot13\,.\,16}{85\cdot6}\right)-\frac{3195}{1880}=+319-1700$$

$$=-1381 \text{ p.s.i. (compression)}$$

Area of Steel Required.

$$At = \frac{237}{120} = 1{\cdot}975 \text{ sq. ins.}$$

Use 63 No. 0·2″ dia. wires at 120,000 p.s.i.
or to utilise cables in which the wires must usually be in even numbers say **4 No.—16 wire cables at 118,000 p.s.i.**

Shear Forces.

1. Due to external loading.

(*a*) Dead load $\frac{352}{144} \,.\, 150 \,.\, 15 = 5{\cdot}5.$

(*b*) Applied load $2{\cdot}0 \,.\, 15$ $\qquad 30{\cdot}0.$

35·5 Kips.

2. Due to cable.

$$\frac{279}{\sqrt{\left(\frac{30 \,.\, 12}{4 \,.\, 8{\cdot}13}\right)^2 + 1}} = 25{\cdot}1 \text{ Kips initially.}$$

and 21·3 Kips finally.

Maximum shear to be resisted.

(*a*) $5{\cdot}5 - 25{\cdot}1 = -19{\cdot}6$ Kips.
(*b*) $35{\cdot}5 - 21{\cdot}3 = +14{\cdot}2$ Kips.

Maximum shear stress.

(*a*) $= \frac{-19{\cdot}6}{32 \,.\, 11} \,.\, 1{\cdot}5 = -84$ p.s.i.

(*b*) $= \frac{+14{\cdot}2}{32 \,.\, 11} \,.\, 1{\cdot}5 = +60$ p.s.i.

Average compressive stress at neutral axis.

initially $\frac{-279}{32 \,.\, 11} = -794$ p.s.i.

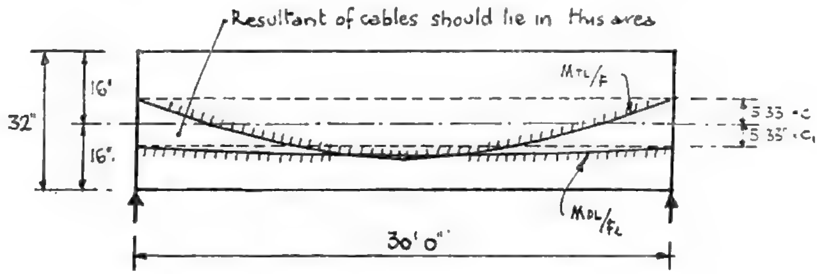
finally $\frac{-237}{32 \,.\, 11} = -674$ p.s.i.

Principal Tensions.

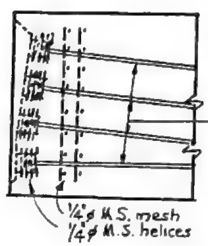
(*a*) $pt = -\frac{794}{2} + \sqrt{\frac{794^2}{4} + 84^2} = +9$ p.s.i. (tension).

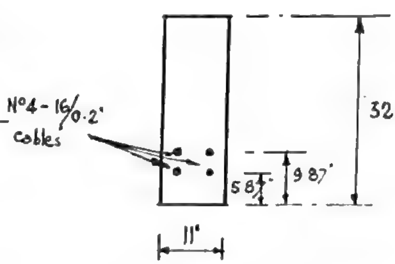
$$(b)\quad pt = -\frac{674}{2} + \sqrt{\frac{674^2}{4} + 60^2} = +5 \text{ p.s.i.}$$

These values are too small to be determined by Mohr's circle and would not be computed in practice as q < allowable pt.



ELEVATION OF BEAM SHEWING LIMITING POSITIONS FOR RESULTANT OF CABLES
(Exagerated vertical scale for clarity)





ELEVATION OF END OF BEAM

CROSS SECTION AT MIDSPAN

Example 2.

Re-design the previous example using precast I sections and with the following stresses :—

Allowable compressive stress at release	=	2,000 p.s.i. (f^1b)
Allowable tensile stress at release	=	0 (f^1t)
Allowable compressive stress at working load	=	1,500 p.s.i. (ft)
Allowable tensile stress at working load	=	0 (fb)

Principal tensile stress = 125 p.s.i. (pt)
Tensile stress in wires at working load = 150,000 p.s.i. (t)
η = 0.85

Moments due to applied load as before.

$$M_1 = 1350 \text{ Kip in.}$$
$$M_2 = 1350 \quad ,, \quad ,,$$
$$M_{AL} = 2700 \quad ,, \quad ,,$$

Assume $M_{DL} = 350$ Kip in.

$$Z_1 = \frac{M_{TL} - \eta\, M_{DL}}{fb - \eta\, f^1 b}$$

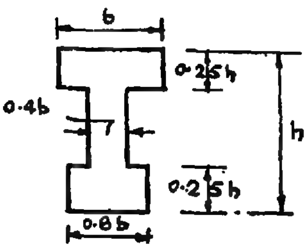
$$= \frac{3050 - 0{\cdot}85 \,.\, 350}{0 + 0{\cdot}85 \,.\, 2000} = \mathbf{1620\ ins.^3}$$

$$Z = \frac{M_{TL} - \eta\, M_{DL}}{\eta\, f^1 t - ft}$$

$$= \frac{3050 - 0{\cdot}85 \,.\, 350}{0 + 1500} = \mathbf{1835\ ins.^3}$$

$$\frac{Z}{Z} = 1{\cdot}13$$

To choose a section we will use the shape factor tables, and keeping in mind practical sizes for manufacture, we see that a section having the following proportions will be suitable.



$$Z = 0{\cdot}890 \frac{bh^2}{6}$$

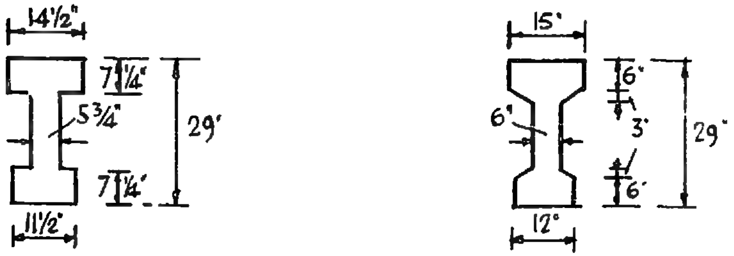
$$Z_1 = 0{\cdot}794 \frac{bh^2}{6}$$

If we let $b = h/2$

$$Z_1 = \frac{0{\cdot}794\, h^3}{12} = 1620 \text{ ins.}^3$$

$$h = 29 \text{ ins.}$$

A section such as at (a) will give the proportion assumed but for practical reasons we will use the equivalent section at (b).



(a) (b)

The properties of section are :—

A	=	285 sq. ins.	Z	=	1860 ins.^{3}
v_1	=	15·35"	Z_1	=	1650 ins.^{3}
v	=	13·65"	ρ^2	=	89·0 ins.^{2}
I	=	25381 ins.^{4}	c	=	5·8"
			c_1	=	6·52"

Actual M_{DL} = 400 Kip ins.

$$F = \frac{M_{TL} - \eta\, M_{DL}}{c + c_1}$$

$$= \frac{3100 - 0{\cdot}85 \,.\, 400}{5{\cdot}8 + 6{\cdot}52} = 224 \text{ Kips}$$

$$F = F/\eta = 224/0{\cdot}85 = 263 \text{ Kips}$$

$$e = c_1 + \frac{M_{DL}}{F_i} = 6{\cdot}52 + \frac{400}{263} = 8{\cdot}04''$$

Check on Stresses due to Bending and Prestress.

At release of cables :—

Bottom fibres :

$$-\frac{F_i}{A}\left(1 + \frac{ev_1}{\rho^2}\right) + \frac{M_{DL}}{Z_1} = f^1b$$

$$= -\frac{263}{285}\left(1 + \frac{8{\cdot}04 \,.\, 15{\cdot}35}{89{\cdot}0}\right) + \frac{400}{1650} = -2210 + 242$$

$$= -1968 \text{ p.s.i.}$$

Top fibres :

$$-\frac{F_i}{A}\left(1 - \frac{ev}{\rho^2}\right) - \frac{M_{DL}}{Z} = f^1t$$

$$-\frac{263}{285}\left(1 - \frac{8{\cdot}04 \,.\, 13{\cdot}65}{89{\cdot}0}\right) - \frac{400}{1860} = +212 - 215$$

$$= -3 \text{ p.s.i.}$$

Under reduced prestress and dead weight of floor :—

Bottom fibres :

$$-\frac{F}{A}\left(1+\frac{ev_1}{\rho^2}\right)+\frac{M_{DL}+M_1}{Z_1}=fb$$

$$-\frac{224}{285}\left(1+\frac{8{\cdot}04\,.\,15{\cdot}35}{89{\cdot}0}\right)+\frac{1750}{1650}=-1880+1060$$
$$=-820 \text{ p.s.i.}$$

Top fibres :

$$-\frac{224}{285}\left(1-\frac{8{\cdot}04\,.\,13{\cdot}65}{89{\cdot}0}\right)-\frac{1750}{1860}=+180-940$$
$$=-760 \text{ p.s.i.}$$

Under reduced prestress and total load :—

Bottom fibres :

$$-\ 1880+\frac{3100}{1650}=-1880+1880=0$$

Top fibres :

$$+\ 180-\frac{3100}{1860}=+180-1670=1490 \text{ p.s.i.}$$

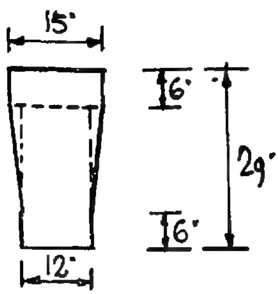
Area of Steel Required.

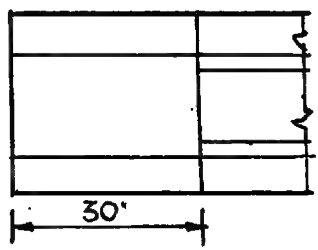
$$At=\frac{224}{150}=1{\cdot}49 \text{ sq. ins.}$$

Use 4 No. 12/0·2″ wire cables.

End Sections.

Make the end section solid for a distance of 30″ at each end.





Shearing Forces.

At support :

(*a*) Due to external loads :

(i)	Dead load	4·95 Kips
(ii)	Applied load	30·0 ,,
		34·95 ,,

(*b*) Due to cables :

(i) Initially $\dfrac{263}{\sqrt{\left(\dfrac{30.12}{4\,.\,7{\cdot}94}\right)^2 + 1}} = 23{\cdot}1$ Kips

(ii) Finally $= 19{\cdot}6$ Kips

Max. shear forces to be resisted :

(1) $4{\cdot}95 - 23{\cdot}1 = -\ 18{\cdot}65$ Kips

(2) $34{\cdot}95 - 19{\cdot}6 = +\ 14{\cdot}85$ Kips

The maximum shearing and principal stresses will occur where the section changes from an I section to a solid section. At this point the shearing forces are :

1. External loads :

(*a*)	Dead load	3·72 Kips
(*b*)	Applied load	25·0 ,,
		28·72 ,,

2. Due to cable (assuming a parabolic form for the cable resultant) :

Initially $\dfrac{23{\cdot}1}{15} \times 12{\cdot}5 = 19{\cdot}25$ Kips

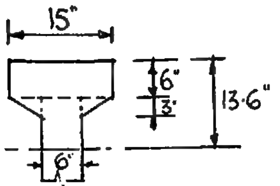
Finally $= 16{\cdot}35$ Kips

Shears to be Resisted.

1. $3{\cdot}72 - 19{\cdot}25 = -\ 15{\cdot}53$ Kips
2. $28{\cdot}72 - 16{\cdot}35 = 12{\cdot}37$ Kips

Shearing Stresses at Neutral Axis.

$$q = \frac{VQ}{b\,I}$$



$$
\begin{aligned}
15\,.\,6\,.\,10{\cdot}6 &= 954\\
8{\cdot}5\,.\,1{\cdot}5\,.\,6{\cdot}6 &= 84\\
7{\cdot}6\,.\,6\,.\,3{\cdot}8 &= 173\\
Q &= 1211 \text{ ins.}
\end{aligned}
$$

1. $$q = -\frac{15{\cdot}53\,.\,1211}{6\,.\,25381} = -\,124 \text{ p.s.i.}$$

2. $$q = \frac{12{\cdot}37\,.\,1211}{6\,.\,25381} = 99 \text{ p.s.i.}$$

Compres ive Stresses at Neutral Axis 2·5′ from Support.

At 2·5′ from the support e

$$= 15{\cdot}35 - \left[15{\cdot}25 - 4\,.\,7{\cdot}94\left(\frac{2{\cdot}5}{30}\right)\left(1-\frac{2{\cdot}5}{30}\right)\right]$$

$$= 2{\cdot}52''$$

At 2·5′ from the support M_{DL}

$$= 4{\cdot}400\,.\left(\frac{2{\cdot}5}{30}\right)\left(1-\frac{2{\cdot}5}{30}\right) = 122 \text{ Kip ins.}$$

At 2·5′ from the support M_{TL}

$$= 4{\cdot}3100\left(\frac{2{\cdot}5}{30}\right)\left(1-\frac{2{\cdot}5}{30}\right) = 946 \text{ Kip ins.}$$

1. For dead load and full prestress :

Bottom flange :

$$-\frac{263}{285}\left(1+\frac{2{\cdot}52\,.\,15{\cdot}35}{89}\right)+\frac{122}{1650} = -1325+740$$

$$= -\,585 \text{ p.s.i.}$$

Top flange :

$$-\frac{263}{285}\left(1-\frac{2{\cdot}52\,.\,13{\cdot}65}{89}\right)-\frac{122}{1860} = +\,563-655$$

$$= -\,92 \text{ p.s.i.}$$

2. For total load and reduced prestress :

Bottom flange :

$$-\,0{\cdot}85\,.\,1325+\frac{946}{1650} = -\,1130+573 = -557 \text{ p.s.i.}$$

Top flange:

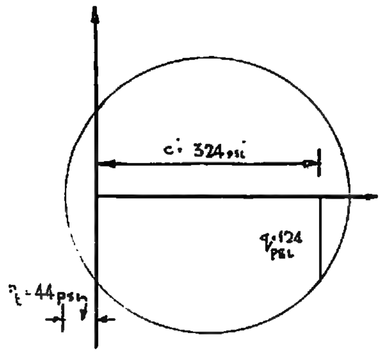
$$+\ 0{\cdot}85\ .\ 563\ -\ \frac{946}{1860}\ =\ +\ \ 477-510\ =\ -\ 33\ \text{p.s.i.}$$

Compressive Stress at Neutral Axis.

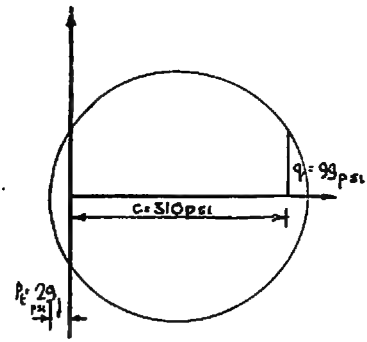
(1) $92\ +\ \dfrac{493}{29}\ .\ 13{\cdot}65\ =\ 324$ p.s.i.

(2) $33\ +\ \dfrac{524}{29}\ .\ 15{\cdot}35\ =\ 310$ p.s.i.

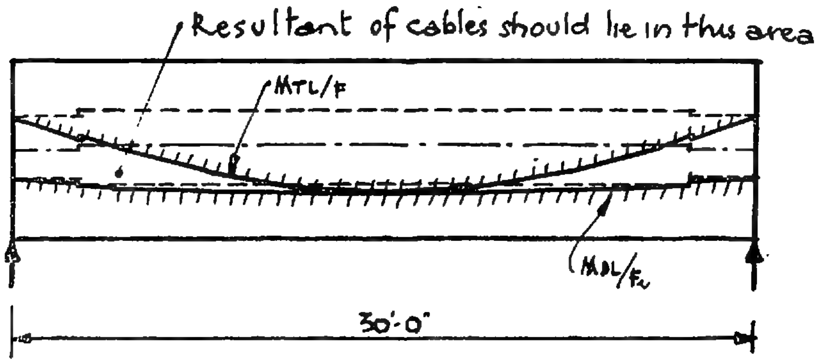
Principal Stresses.



MOHRS CIRCLE FOR (1)



MOHRS CIRCLE FOR (2)



ELEVATION OF BEAM SHEWING LIMITING POSITIONS FOR RESULTANT OF CABLES

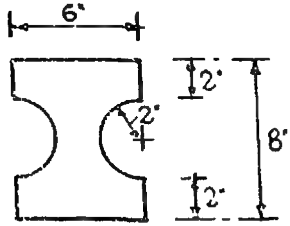
(Exaggerated vertical scale for clarity)

Example 3.

A precast floor is required to carry 150 lbs. per sq. foot on a span of 20′ 0″. Using the standardised section given below, determine the prestressing force and its eccentricity if the unit is to be designed so that it may be lifted either way up.

Allowable stresses :

Maximum compressive stress on release	$\not>$ 1750 p.s.i.
Maximum compressive stress under working load	$\not>$ 1500 p.s.i.
Maximum tension allowable when unit is lifted wrong way up	$\not>$ 100 p.s.i.
Tension under working load	0
Principal tension	$\not>$ 120 p.s.i.
Steel stress at working load	$\not>$ 120,000 p.s.i.
	$\eta = 0{\cdot}85$



STANDARD PRECAST FLOOR UNIT

Properties of Section.

$$A = 35{\cdot}4 \text{ in.}^2; \quad v = v_1 = 4{\cdot}0''$$
$$I = 243{\cdot}4 \text{ in.}^4; \quad Z = Z_1 = 60{\cdot}9 \text{ in.}^3;$$
$$\rho^2 = 6{\cdot}88 \text{ ins.}^2$$

Moment Due to Self Weight.

$$\frac{35{\cdot}4}{144} \,.\, 150 \,.\, 20^2 \,.\, 1{\cdot}5 \quad = 22{\cdot}2 \text{ Kip ins.}$$

Moment Due to Applied Load.

$$\frac{150}{2} \,.\, 20^2 \,.\, 1{\cdot}5 \quad = 45{\cdot}0 \text{ Kip ins.}$$

then $M_1 = +\,45{\cdot}0 + 22{\cdot}2 = +$ **67·2 Kip ins.**

$M_2 = -$ **22·2 Kip ins.**

Required Final Values of Stresses Due to Prestressing.

$$\sigma^1{}_p = fb - \frac{M_1}{Z_1} = 0 - \frac{67{\cdot}2}{60{\cdot}9} = -\ 1105 \text{ p.s.i.}$$

$$< \frac{M_2}{Z_1} + ft = -\frac{22{\cdot}2}{60{\cdot}9} - 1500 = -1865 \text{ p.s.i.}$$

$$\sigma_p = \frac{M_1}{Z} + ft = \frac{67{\cdot}2}{60{\cdot}9} - 1500 = -\ 395 \text{ p.s.i.}$$

$$< fb + \frac{M_2}{Z} = +\ 100-365 = -\ 265 \text{ p.s.i.}$$

Required Final Value of Prestressing Force.

$$F = -\ (v\sigma^1{}_p + v_1\sigma_p)\ A/h = -\ (+4\ .\ 1{\cdot}105 + 4\ .\ 0{\cdot}265)\ \frac{35{\cdot}4}{8}$$

$$= -\ 24{\cdot}3 \text{ Kips}$$

$$F_i = -\ 28{\cdot}5 \text{ Kips}$$

Area of Steel Required.

$At = \dfrac{24{\cdot}3}{120} =$ **0·203 sq. ins.** Use 7 No. 0·2″ dia. wires.

Required Eccentricity.

$$e = \frac{\sigma^1{}_p - \sigma_p}{v\sigma^1{}_p + v_1\sigma_p}\ \rho^2 = \frac{-265 + 1105}{+5480}\ .\ 6{\cdot}88$$

$$= \mathbf{1{\cdot}05''}$$

Stresses Due to Prestressing.

Initially

$$-\frac{28{\cdot}5}{35{\cdot}4}\left(1 \pm \frac{4\ .\ 1{\cdot}05}{6{\cdot}88}\right) = \begin{array}{l} -\ 1300 \text{ p.s.i. bottom fibre and} \\ -\ 314 \text{ p.s.i. top fibre.} \end{array}$$

Finally

$$-\frac{0{\cdot}85\ .\ 28{\cdot}5}{35{\cdot}4}\left(1 \pm \frac{4\ .\ 1{\cdot}05}{6{\cdot}88}\right) = \begin{array}{l} -1105 \text{ p.s.i. bottom} \\ \text{fibre and} \\ -267 \text{ p.s.i. top fibre.} \end{array}$$

Stresses Due to Prestressing and Loading.

Dead Load + Prestress.

Initially.

Top fibre : $-\ 314 - \dfrac{22\cdot2}{60\cdot9} = -\ 679$ p.s.i.

Bottom fibre : $-\ 1300 + \dfrac{22\cdot2}{60\cdot9} = -\ 935$ p.s.i.

Finally.

Top fibre : $-\ 267 - \dfrac{22\cdot2}{60\cdot9} = -\ 632$ p.s.i.

Bottom fibre : $-\ 1105 + \dfrac{22\cdot2}{60\cdot9} = -\ 740$ p.s.i.

Total Load + Prestress.

Top fibre : $-\ 267 - \dfrac{67\cdot2}{60\cdot9} = -\ 1372$ p.s.i.

Bottom fibre : $-\ 1105 + \dfrac{67\cdot2}{60\cdot9} = 0$

Dead Load Acting in Reverse Direction + Prestress.

Initially.

Top fibre : $-\ 314 + \dfrac{22\cdot2}{60\cdot9} = +\ 51$ p.s.i.

Bottom fibre : $-\ 1300 - \dfrac{22\cdot2}{60\cdot9} = -\ 1665$ p.s.i.

Finally.

Top fibre : $-\ 267 + \dfrac{22\cdot2}{60\cdot9} = +\ 98$ p.s.i.

Bottom fibre : $-\ 1105 - \dfrac{22\cdot2}{60\cdot9} = -\ 1470$ p.s.i.

Shearing Force.

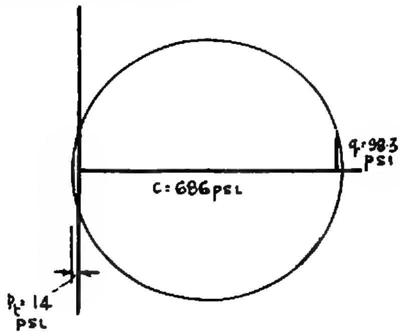
$$\left(\frac{35\cdot4}{144} \times 150\right) + \frac{150}{2} = 112 \text{ lbs./f.r.}$$

$$V = 112 . 10 = 1120 \text{ lbs.}$$
$$Q = 42{\cdot}6 \text{ in.}^3$$
$$q = \frac{1120 . 42{\cdot}6}{2 . 243{\cdot}4} = 98{\cdot}3 \text{ p.s.i.}$$

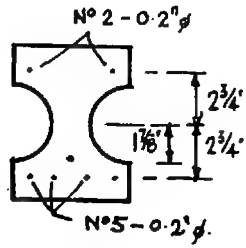
Compressive stress at neutral axis (final value).

$$c = \frac{1105 + 267}{2} = \frac{1372}{2} = 686 \text{ p.s.i.}$$

Principal Tension.



MOHRS CIRCLE



CROSS SECTION SHEWING POSITION OF WIRES.

BIBLIOGRAPHY.

The numbers in parenthesis indicate the references from the text.

(1) MARSHALL, G.—"End Anchorage and Bond Stress in Prestressed Concrete." "Magazine of Concrete Research," London, Dec., 1949, No. 3, pp. 123-129.

(2) RITTER, M. and LARDY, P.—"Vorgespannter Beton." 1st Edition, Zurich, 1945, pp. 118, or translated into French by Delarue, J., "Le Béton Précontraint," Paris, Dunod, 1947, pp. vi., 90.

(3) WEINBERG, V. and KRAVTZOFF, W.—"Le Béton Précontraint" (Premier Part, Mèthodes de calcul). 1st Edition, Paris, Dunod, 1947, pp. vi., 51.

(4) GUYON, Y.—"Poutres en Béton Précontraint de Section Uniform à Cables Releves." Circular Serie J, No. 9, 1947, Inst. Tech. du Bat. et T.P., pp. 24.

(5) BARETS, J.—"Méthode pour la détermination rapide des diagrammes d'efforts tranchant réduits optima dans les poutres en béton précontraint à câbles réleves." Travaux, Paris, Vol. 32, Oct., 1948, pp. 541-544.

(6) ROBERT, E. and LEBELLE, P.—"Prestressed Concrete Reservoir at Orleans." C.A.C.A., London, Library Translation Cj. 10, 1949.

(7) GUYON, Y.—"Étude sur les poutres continues et sur certaines systèms hyperstatiques en béton précontraint." Circular Serie J, No. 8, 1945, Inst. Tech. du Bat. et T.P., pp. 36.

(8) MÖRSCH, E.—"Determination of the Ultimate Bending Strength of Prestressed Concrete Beams." C.A.C.A., London, Library Translation Cj. 32, 1951.

(9) EVANS, R. H.—"Plastic Theories for the Ultimate Strength of R.C. Beams." Jnl. Inst. Civil Engineers, No. 2, 1943-44, Dec., 1943, pp. 98-121.

(10) BAKER, A. L. L.—"Ultimate Strength in Bending of Prestressed Concrete Beams." Reinforced Concrete Association, London. Reinforced Concrete Review, Vol. II., No. 6, July, 1951, p. 343.

(11) GUERIN, P.—"Le calcul du béton arme a la rupture." 1st Edition, Paris, Librairie de L'enseignment Technique, 1949, pp. 335.

Books and Articles on Prestressed Concrete other than those Mentioned in the Text.

MAGNEL, G.—"Prestressed Concrete." 2nd Edition, London, Concrete Publications, Ltd., 1950, pp. vi., 300.

ABELES, P. W.—"Principles and Practice of Prestressed Concrete." 1st Edition, London, Crosby Lockwood & Son, Ltd., 1949, pp. 124.

BILLIG, R.—"Prestressed Reinforced Concrete." 2nd Edition, London, Knapp, Drewett & Sons, Ltd., 1944, pp. 248.

MAUTNER, K. W.—"Prestressed Concrete in Structures of Annular Cross Section." "Structural Engineer," London, 1945, Vol. 23, pp. 117-162.

LALANDE, M.—"The Variety and Application of Prestressed Concrete." Cement and Concrete Association, London, Library Translation Cj. 7, 1949.

FREYSSINET, E.—"Une révolution dans les techniques du Béton." 1st Edition, Paris, Librarie de L'enseignment Technique, Leon Eyrolles, 1936. Also given in part as a lecture to the Institution of Structural Engineers and published as : "A Revolution in the Technique of the Utilization of Concrete." "Structural Engineer," 1936, Vol 14, pp. 242-259. "Prestressed Concrete : Principles and Applications." Journal Institute Civil Engineers, 1949, Vol. 33, pp. 331-380.

GUYON, Y.—"Théorie des poutres et dalles en béton précontraint." Circular Serie J, No. 1, 1944. Institute Technique du Bâtiment et Travaux Publics, p. 14. "Poutres et dalles précontraint." Circular Serie J, No. 3, 1944, Inst. Tech. du Bat. et T.P., p. 50.

LOSSIER, H.—"The Self Stressing of Concrete by Expanding Cements." C.A.C.A., London, Library Translation Cj. 23, 1950.

Books on Concrete Construction, etc.

MURDOCH, L. J.—"Concrete Materials and Practice." 1st Edition, London, Arnold & Co., 1948, pp. vi., 328.

WYNN, A. E.—"Design and Construction of Formwork for Concrete Structure." 2nd Edition, London, Concrete Publications, Ltd., 1947, pp. 302.

COLLINS, A. R.—"High Strength Concrete." Reinforced Concrete Review, London, Vol. II. (i), Jan., 1950, pp. 29-47.

SPARKS, F.—"Control of Variations in Quality of Concrete and its Effect on Mix Proportions." Reinforced Concrete Review, London, Vol. I. (13), October, 1949, pp. 543-570.

SAUL, A. G. A.—"Principles Underlying the Steam Curing of Concrete at Atmospheric Pressure." C.A.C.A., London, Research Note, Rp., 1950, pp. 18.

APPENDIX

TABLES GIVING THE GEOMETRIC PROPERTIES OF VARIOUS SECTIONS

TABLES 1, 2 & 3 REFER TO I & BOX SECTIONS

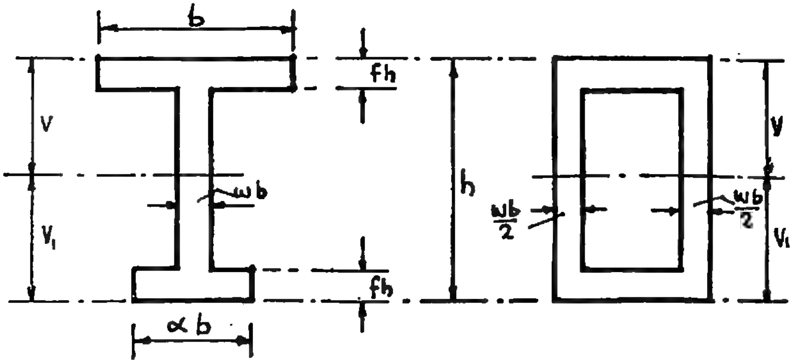
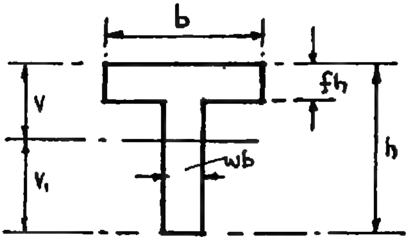


TABLE 4 REFERS TO T SECTIONS



Z refers to the Section Modulus of the upper face as drawn.

Z_1 refers to the Section Modulus of the lower face as drawn.

TABLE 1.

α	w	f	Area $\times bh$	v_1 $\times h$	v $\times h$	I $\times \frac{bh^3}{12}$	Z $\times \frac{bh^3}{6}$	Z_1 $\times \frac{bh^2}{6}$
1·0	0·1	0·05	0·19	0·5	0·5	0·343	0·343	0·343
		0·10	0·28			0·539	0·539	0·539
		0·15	0·37			0·691	0·691	0·691
		0·20	0·46			0·805	0·805	0·805
		0·25	0·55			0·887	0·887	0·887
	0·2	0·05	0·28			0·416	0·416	0·416
		0·10	0·36			0·590	0·590	0·590
		0·15	0·44			0·725	0·725	0·725
		0·20	0·52			0·827	0·827	0·827
		0·25	0·60			0·900	0·900	0·900
	0·3	0·05	0·37			0·488	0·488	0·488
		0·10	0·44			0·641	0·641	0·641
		0·15	0·51			0·760	0·760	0·760
		0·20	0·58			0·849	0·849	0·849
		0·25	0·65			0·912	0·912	0·912
	0·4	0·05	0·46			0·562	0·562	0·562
		0·10	0·52			0·693	0·693	0·693
		0·15	0·58			0·794	0·794	0·794
		0·20	0·64			0·870	0·870	0·870
		0·25	0·70			0·925	0·925	0·925
	0·5	0·05	0·55			0·635	0·635	0·635
		0·10	0·60			0·744	0·744	0·744
		0·15	0·65			0·829	0·829	0·829
		0·20	0·70			0·892	0·892	0·892
		0·25	0·75			0·938	0·938	0·938
0·9	0·1	0·05	0·185	0·510	0·490	0·337	0·344	0·330
		0·10	0·270	0·516	0·484	0·515	0·532	0·499
		0·15	0·355	0·518	0·482	0·652	0·677	0·630
		0·20	0·440	0·518	0·482	0·757	0·786	0·732
		0·25	0·525	0·518	0·482	0·846	0·878	0·816
	0·2	0·05	0·275	0·510	0·490	0·404	0·412	0·397
		0·10	0·350	0·513	0·487	0·560	0·575	0·545
		0·15	0·425	0·516	0·484	0·685	0·709	0·664
		0·20	0·500	0·516	0·484	0·778	0·803	0·756
		0·25	0·575	0·517	0·483	0·855	0·887	0·828
	0·3	0·05	0·365	0·507	0·493	0·477	0·485	0·471
		0·10	0·430	0·511	0·489	0·611	0·625	0·598
		0·15	0·495	0·513	0·487	0·721	0·740	0·704
		0·20	0·560	0·514	0·486	0·809	0·834	0·708
		0·25	0·625	0·515	0·485	0·859	0·886	0·835
	0·4	0·05	0·455	0·506	0·496	0·556	0·560	0·548
		0·10	0·510	0·508	0·492	0·661	0·673	0·651
		0·15	0·565	0·512	0·488	0·755	0·773	0·738
		0·20	0·620	0·513	0·487	0·831	0·852	0·810
		0·25	0·675	0·514	0·486	0·884	0·910	0·862
	0·5	0·05	0·545	0·505	0·495	0·616	0·623	0·610
		0·10	0·590	0·507	0·493	0·714	0·725	0·705
		0·15	0·635	0·511	0·489	0·790	0·800	0·773
		0·20	0·680	0·512	0·488	0·843	0·863	0·824
		0·25	0·725	0·513	0·487	0·896	0·921	0·874

TABLE 2.

α	w	f	Area	v_1	v	I	Z	Z_1
			$\times bh$	$\times h$	$\times h$	$\times \frac{bh^3}{12}$	$\times \frac{bh^2}{6}$	$\times \frac{bh^2}{6}$
0·8	0·1	0·05	0·180	0·527	0·473	0·315	0·333	0·297
		0·10	0·260	0·535	0·465	0·485	0·522	0·454
		0·15	0·340	0·538	0·462	0·615	0·665	0·572
		0·20	0·420	0·538	0·462	0·720	0·779	0·670
		0·25	0·520	0·538	0·462	0·785	0·853	0·732
	0·2	0·05	0·270	0·518	0·482	0·383	0·397	0·370
		0·10	0·340	0·526	0·474	0·544	0·574	0·517
		0·15	0·410	0·532	0·468	0·650	0·695	0·612
		0·20	0·480	0·533	0·467	0·742	0·795	0·697
		0·25	0·550	0·535	0·465	0·799	0·860	0·747
	0·3	0·05	0·360	0·514	0·486	0·450	0·464	0·438
		0·10	0·420	0·523	0·477	0·590	0·618	0·564
		0·15	0·480	0·527	0·473	0·688	0·728	0·653
		0·20	0·540	0·530	0·470	0·764	0·813	0·721
		0·25	0·600	0·532	0·468	0·823	0·880	0·774
	0·4	0·05	0·450	0·512	0·488	0·532	0·543	0·518
		0·10	0·500	0·518	0·482	0·650	0·675	0·627
		0·15	0·550	0·524	0·476	0·720	0·757	0·688
		0·20	0·600	0·526	0·474	0·786	0·831	0·748
		0·25	0·650	0·529	0·471	0·838	0·890	0·794
	0·5	0·05	0·540	0·510	0·490	0·604	0·616	0·592
		0·10	0·580	0·516	0·484	0·691	0·714	0·670
		0·15	0·620	0·522	0·478	0·767	0·793	0·735
		0·20	0·660	0·525	0·475	0·808	0·851	0·770
		0·25	0·700	0·528	0·472	0·844	0·895	0·800
0·7	0·1	0·05	0·175	0·540	0·460	0·300	0·326	0·278
		0·10	0·250	0·554	0·446	0·463	0·518	0·418
		0·15	0·325	0·558	0·442	0·581	0·658	0·520
		0·20	0·400	0·560	0·440	0·666	0·757	0·595
		0·25	0·475	0·560	0·440	0·736	0·836	0·658
	0·2	0·05	0·265	0·526	0·474	0·375	0·395	0·355
		0·10	0·330	0·541	0·459	0·515	0·562	0·476
		0·15	0·395	0·548	0·452	0·618	0·683	0·565
		0·20	0·460	0·552	0·448	0·695	0·775	0·630
		0·25	0·525	0·553	0·447	0·751	0·842	0·682
	0·3	0·05	0·355	0·520	0·480	0·448	0·467	0·432
		0·10	0·410	0·533	0·467	0·560	0·600	0·525
		0·15	0·485	0·542	0·458	0·652	0·712	0·602
		0·20	0·520	0·547	0·453	0·719	0·795	0·656
		0·25	0·575	0·549	0·451	0·764	0·847	0·698
	0·4	0·05	0·445	0·516	0·484	0·531	0·548	0·515
		0·10	0·490	0·527	0·473	0·616	0·652	0·585
		0·15	0·535	0·536	0·464	0·687	0·740	0·642
		0·20	0·580	0·542	0·458	0·749	0·818	0·693
		0·25	0·625	0·546	0·454	0·778	0·857	0·712
	0·5	0·05	0·535	0·514	0·486	0·593	0·610	0·577
		0·10	0·570	0·523	0·477	0·667	0·698	0·637
		0·15	0·605	0·531	0·469	0·721	0·770	0·679
		0·20	0·640	0·538	0·462	0·763	0·826	0·709
		0·25	0·675	0·541	0·459	0·789	0·860	0·729

TABLE 3.

α	w	f	Area $\times bh$	v_1 $\times h$	v $\times h$	I $\times \frac{bh^3}{12}$	Z $\times \frac{bh^2}{6}$	Z_1 $\times \frac{bh^2}{6}$
0·6	0·1	0·05	0·170	0·555	0·445	0·280	0·315	0·253
		0·10	0·240	0·575	0·425	0·424	0·498	0·369
		0·15	0·310	0·582	0·418	0·535	0·640	0·458
		0·20	0·380	0·584	0·416	0·618	0·743	0·529
		0·25	0·450	0·584	0·416	0·666	0·801	0·570
	0·2	0·05	0·260	0·536	0·464	0·365	0·394	0·341
		0·10	0·320	0·556	0·444	0·473	0·534	0·426
		0·15	0·380	0·567	0·433	0·574	0·663	0·506
		0·20	0·440	0·573	0·426	0·643	0·755	0·562
		0·25	0·500	0·575	0·425	0·682	0·802	0·594
	0·3	0·05	0·350	0·527	0·473	0·440	0·465	0·418
		0·10	0·400	0·545	0·455	0·531	0·584	0·487
		0·15	0·450	0·557	0·443	0·608	0·685	0·545
		0·20	0·500	0·564	0·435	0·663	0·762	0·587
		0·25	0·550	0·568	0·432	0·706	0·818	0·621
	0·4	0·05	0·440	0·522	0·478	0·506	0·528	0·485
		0·10	0·480	0·538	0·462	0·581	0·629	0·540
		0·15	0·520	0·540	0·451	0·642	0·712	0·585
		0·20	0·560	0·557	0·442	0·686	0·776	0·615
		0·25	0·600	0·562	0·438	0·721	0·824	0·642
	0·5	0·05	0·530	0·517	0·483	0·580	0·601	0·562
		0·10	0·560	0·533	0·467	0·635	0·680	0·596
		0·15	0·590	0·543	0·457	0·689	0·754	0·636
		0·20	0·620	0·552	0·448	0·717	0·800	0·649
		0·25	0·650	0·557	0·443	0·733	0·828	0·658
0·5	0·1	0·05	0·165	0·573	0·427	0·264	0·309	0·231
		0·10	0·230	0·597	0·403	0·384	0·476	0·321
		0·15	0·295	0·609	0·391	0·479	0·614	0·394
		0·20	0·360	0·612	0·388	0·547	0·706	0·447
		0·25	0·425	0·612	0·388	0·582	0·766	0·475
	0·2	0·05	0·255	0·546	0·454	0·339	0·375	0·310
		0·10	0·310	0·573	0·427	0·437	0·512	0·381
		0·15	0·365	0·588	0·412	0·528	0·643	0·450
		0·20	0·420	0·596	0·404	0·585	0·725	0·492
		0·25	0·475	0·599	0·401	0·622	0·776	0·519
	0·3	0·05	0·345	0·535	0·465	0·421	0·453	0·394
		0·10	0·390	0·558	0·442	0·498	0·564	0·446
		0·15	0·435	0·574	0·426	0·552	0·649	0·482
		0·20	0·480	0·583	0·417	0·612	0·735	0·525
		0·25	0·525	0·589	0·411	0·637	0·777	0·542
	0·4	0·05	0·435	0·528	0·472	0·491	0·521	0·464
		0·10	0·470	0·548	0·452	0·546	0·604	0·498
		0·15	0·505	0·563	0·437	0·609	0·697	0·541
		0·20	0·540	0·575	0·425	0·642	0·757	0·558
		0·25	0·575	0·582	0·418	0·652	0·779	0·561
	0·5	0·05	0·525	0·523	0·477	0·561	0·588	0·536
		0·10	0·550	0·541	0·459	0·612	0·666	0·565
		0·15	0·575	0·556	0·444	0·632	0·712	0·568
		0·20	0·600	0·568	0·432	0·663	0·768	0·584
		0·25	0·625	0·575	0·425	0·676	0·796	0·588

TABLE 4.

α	w	f	Area $\times bh$	v_1 $\times h$	v $\times h$	I $\times \frac{bh^3}{12}$	Z $\times \frac{bh^2}{6}$	Z_1 $\times \frac{bh^2}{6}$
	0·1	0·05	0·145	0·648	0·352	0·183	0·260	0·141
		0·10	0·190	0·713	0·297	0·220	0·370	0·152
		0·15	0·235	0·745	0·255	0·226	0·443	0·152
		0·20	0·280	0·757	0·243	0·230	0·475	0·152
		0·25	0·325	0·760	0·240	0·231	0·480	0·152
	0·2	0·05	0·240	0·579	0·421	0·289	0·343	0·250
		0·10	0·280	0·628	0·372	0·347	0·466	0·276
		0·15	0·320	0·659	0·341	0·364	0·533	0·276
		0·20	0·360	0·678	0·322	0·378	0·587	0·279
		0·25	0·400	0·687	0·312	0·381	0·612	0·278
	0·3	0·05	0·335	0·550	0·450	0·385	0·428	0·350
		0·10	0·370	0·585	0·415	0·434	0·523	0·371
		0·15	0·405	0·610	0·390	0·472	0·606	0·387
		0·20	0·440	0·628	0·372	0·492	0·662	0·392
		0·25	0·475	0·640	0·360	0·500	0·695	0·392
	0·4	0·05	0·430	0·534	0·466	0·462	0·496	0·433
		0·10	0·460	0·560	0·440	0·524	0·596	0·468
		0·15	0·485	0·585	0·415	0·553	0·667	0·473
		0·20	0·520	0·592	0·408	0·583	0·713	0·493
		0·25	0·550	0·602	0·398	0·595	0·748	0·494
	0·5	0·05	0·525	0·523	0·477	0·565	0·592	0·540
		0·10	0·550	0·541	0·459	0·613	0·668	0·567
		0·15	0·575	0·556	0·444	0·636	0·716	0·572
		0·20	0·600	0·567	0·433	0·662	0·760	0·584
		0·25	0·625	0·575	0·425	0·677	0·797	0·589

A.E.S.D. Printed Pamphlets and Other Publications in Stock.

An up-to-date list of A.E.S.D. pamphlets in stock is obtainable on application to the Editor, *The Draughtsman*, 96 St. George's Square, London, S.W.1.

A similar list is also published in *The Draughtsman* twice a year.

Readers are asked to consult this list before ordering pamphlets published in previous sessions.

List of A.E.S.D. Data Sheets.

1. Safe Load on Machine-Cut Spur Gears.
2. Deflection of Shafts and Beams
3. Deflection of Shafts and Beams (Instruction Sheet) } Connected.
4. Steam Radiation Heating Chart.
5. Horse-Power of Leather Belts, etc.
6. Automatic Brakes (Axle Brakes)
7. Automobile Brakes (Transmission Brakes) } Connected.
8. Capacities of Bucket Elevators.
9. Valley Angle Chart for Hoppers and Chutes.
10. Shafts up to 5½-in. diameter, subjected to Twisting and Combined Bending and Twisting.
11. Shafts, 5¾ to 26 inch diameter, subjected to Twisting and Combined Bending and Twisting.
12. Ship Derrick Booms.
13. Spiral Springs (Diameter of Rd. or Sq. Wire).
14. Spiral Springs (Compression).
15. Automobile Clutches (Cone Clutches).
16. ,, ,, (Plane Clutches).
17. Coil Friction for Belts, etc.
18. Internal Expanding Brakes. Self-Balancing Brake Shoes (Force Diagram)
19. Internal Expanding Brakes. Angular Proportions for Self-Balancing. } Connected.
20. Referred Mean Pressure Cut-Off, etc.
21. Particulars for Balata Belt Drives.
22. ⅞″ Square Duralumin Tubes as Struts.
23. 1″ ,, ,, ,, ,,
24. ¾″ Sq. Steel Tubes as Struts (30 ton yield).
25. ⅞″ ,, ,, ,, (30 ton yield).
26. 1″ ,, ,, ,, (30 ton yield).
27. ¾″ ,, ,, ,, (40 ton yield).
28. ⅞″ ,, ,, ,, (40 ton yield).
29. 1″ ,, ,, ,, (40 ton yield).
30. Moments of Inertia of Built-up Sections (Tables)
31. Moments of Inertia of Built-up Sections (Instructions and Examples) } Connected.
32. Reinforced Concrete Slabs (Line Chart)
33. Reinforced Concrete Slabs (Instructions and Examples) } Connected.
34. Capacity and Speed Chart for Troughed Band Conveyors.
35. Screw Propeller Design (Sheet 1, Diameter Chart)
36. ,, ,, ,, (Sheet 2, Pitch Chart)
37. ,, ,, ,, (Sheet 3, Notes and Examples) } Connected.
38. Open Coil Conical Springs.
39. Close Coil Conical Springs.
40. Trajectory Described by Belt Conveyors (Revised).
41. Metric Equivalents.
42. Useful Conversion Factors.
43. Torsion of Non-Circular Shafts.
44. Railway Vehicles on Curves.
46. Coned Plate Developments.
47. Solution of Triangles (Sheet 1, Right Angles).
48. Solution of Triangles (Sheet 2, Oblique Angles).
49. Relation between Length, Linear Movement and Angular Movement of Lever (Diagram and Notes).
50. ,, ,, ,, ,, ,, ,, (Chart).
51. Helix Angle and Efficiency of Screws and Worms.
52. Approximate Radius of Gyration of Various Sections.

53. Helical Spring Graphs (Round Wire).
54. ,, ,, ,, (Round Wire). Connected.
55. ,, ,, ,, (Square Wire).
56. Relative Value of Welds to Rivets.
58. Graphs for Strength of Rectangular Flat Plates of Uniform Thickness.
59. Graphs for Deflection of Rectangular Flat Plates of Uniform Thickness.
60. Moment of Resistance of Reinforced Concrete Beams.
61. Deflection of Leaf Spring.
62. Strength of Leaf Spring.
63. Chart Showing Relationship of Various Hardness Tests.
64. Shaft Horse Power and Proportions of Worm Gear.
65. Ring with Uniform Internal Load (Tangential Strain) Connected.
66. Ring with Uniform Internal Load (Tangential Stress)
67. Hub Pressed on to Steel Shaft. (Maximum Tangential Stress at Bore of Hub).
86. Hub Pressed on to Steel Shaft. (Radial Gripping Pressure between Hub and Shaft).
69. Rotating Disc (Steel) Tangential Strain. Connected.
70. ,, ,, ,, Stress.
71. Ring with Uniform External Load, Tangential Strain. Connected.
72. ,, ,, ,, ,, ,, Stress.
73. Viscosity Temperature Chart for Converting Commercial to Absolute Viscosities. Connected.
74. Journal Friction on Bearings.
75. Ring Oil Bearings.
76. Shearing and Bearing Values for High Tensile Structural Steel Shop Rivets, in accordance with B.S.S. No. 548/1934.
78. Velocity of Flow in Pipes for a Given Delivery. Connected.
79. Delivery of Water in Pipes for a Given Head.
80. (See No. 105).
81. Involute Toothed Gearing Chart.
83. Variation of Suction Lift and Temperature for Centrifugal Pumps.
89. Curve Relating Natural Frequency and Deflection. Connected.
90. Vibration Transmissibility Curved or Elastic Suspension.
91. Instructions and Examples in the Use of Data Sheets, Nos. 89 and 90.
92. Pressure on Sides of Bunker.
93-4-5-6-7. Rolled Steel Sections.
98-99-100. Boiler Safety Valves.
102. Pressure Required for Blanking and Piercing.
103. Punch and Die Clearances for Blanking and Piercing.
104. Nomograph for Valley Angles of Hoppers and Chutes.
105. Permissible Working Stresses in Mild Steel Struts to B.S. 449, 1948.

(Data Sheets are 3d to Members, 6d to others, post free).

Orders for Pamphlets and Data Sheets to be sent to the Editor, *The Draughtsman*, cheques and orders being crossed "A.E.S.D."